Solving Algebraic Equations

1)

For what value of k does the equation $x^2 - 5x + (k + 6) = 0$ have equal roots? **3**

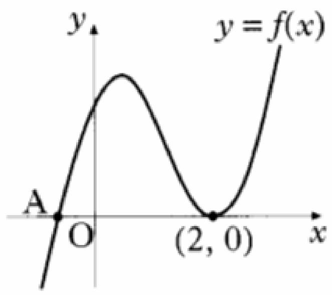
2)

(a) Given that $x + 2$ is a factor of $2x^3 + x^2 + kx + 2$, find the value of k. **3**

(b) Hence solve the equation $2x^3 + x^2 + kx + 2 = 0$ when k takes this value. **2**

3)

The diagram shows part of the graph of the curve with equation $y = 2x^3 - 7x^2 + 4x + 4$.



(a) Find the x-coordinate of the maximum turning point. **5**

(b) Factorise $2x^3 - 7x^2 + 4x + 4$. **3**

(c) State the coordinates of the point A and hence find the values of x for which $2x^3 - 7x^2 + 4x + 4 < 0$. **2**

4)

Show that the equation $(1 - 2k)x^2 - 5kx - 2k = 0$ has real roots for all integer values of k. **5**

5)

$f(x) = 6x^3 - 5x^2 - 17x + 6$.

(a) Show that $(x - 2)$ is a factor of $f(x)$.

(b) Express $f(x)$ in its fully factorised form. **4**

6)

$f(x) = x^3 - x^2 - 5x - 3$.

(a) (i) Show that $(x + 1)$ is a factor of $f(x)$.

(ii) Hence or otherwise factorise $f(x)$ fully. **5**

(b) One of the turning points of the graph of $y = f(x)$ lies on the x-axis. Write down the coordinates of this turning point. **1**

7)

(*a*) Write $x^2 - 10x + 27$ in the form $(x + b)^2 + c$. **2**

(*b*) Hence show that the function $g(x) = \frac{1}{3}x^3 - 5x^2 + 27x - 2$ is always increasing. **4**

8)

Prove that the roots of the equation $2x^2 + px - 3 = 0$ are real for all values of p. **4**

9)

A function f is defined by the formula $f(x) = 2x^3 - 7x^2 + 9$ where x is a real number.

(*a*) Show that $(x - 3)$ is a factor of $f(x)$, and hence factorise $f(x)$ fully. **5**

(*b*) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x- and y-axes. **2**

(*c*) Find the greatest and least values of f in the interval $-2 \leq x \leq 2$. **5**

10)

(*a*) Show that $x = -1$ is a solution of the cubic equation $x^3 + px^2 + px + 1 = 0$. **1**

(*b*) Hence find the range of values of p for which all the roots of the cubic equation are real. **7**

11)

(*a*) Express $2x^2 + 4x - 3$ in the form $a(x + b)^2 + c$. **3**

(*b*) Write down the coordinates of the turning point on the parabola with equation $y = 2x^2 + 4x - 3$. **1**

12)

Find the value of k such that the equation $kx^2 + kx + 6 = 0$, $k \neq 0$, has equal roots. **4**

13)

Find the range of values of k such that the equation $kx^2 - x - 1 = 0$ has no real roots. **4**

Solving Trigonometric Equations

1)

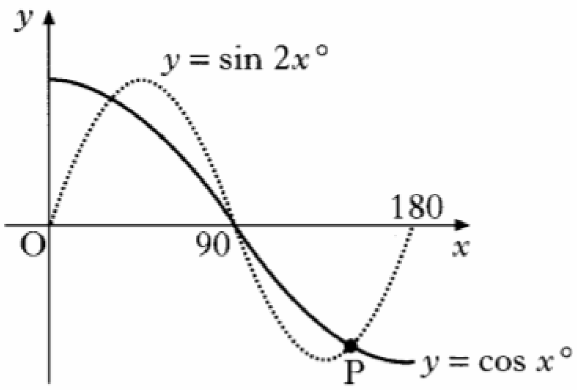
Solve the equation $\cos 2x° + 2\sin x° = \sin^2 x°$ in the interval $0 \le x < 360$. **5**

2)

(*a*) Solve the equation $\sin 2x° - \cos x° = 0$ in the interval $0 \le x \le 180$. **4**

(*b*) The diagram shows parts of two trigonometric graphs, $y = \sin 2x°$ and $y = \cos x°$.

Use your solutions in (*a*) to write down the coordinates of the point P. **1**



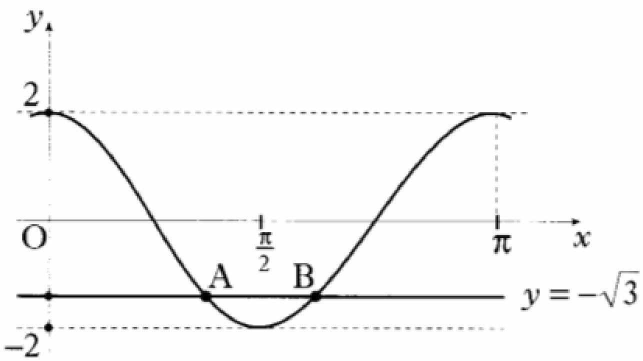
3)

The diagram shows the graph of a cosine function from 0 to π.

(*a*) State the equation of the graph. **1**

(*b*) The line with equation $y = -\sqrt{3}$ intersects this graph at points A and B.

Find the coordinates of B. **3**



4)

Solve the equation $3\cos(2x) + 10\cos(x) - 1 = 0$ for $0 \leq x \leq \pi$, correct to 2 decimal places. **5**

5)

Find all the values of x in the interval $0 \leq x \leq 2\pi$ for which $\tan^2(x) = 3$. **4**

6)

(*a*) Express $3\cos(x°) + 5\sin(x°)$ in the form $k\cos(x° - a°)$ where $k > 0$ and $0 \leq a \leq 90$. **4**

(*b*) Hence solve the equation $3\cos(x°) + 5\sin(x°) = 4$ for $0 \leq x \leq 90$. **3**

7)

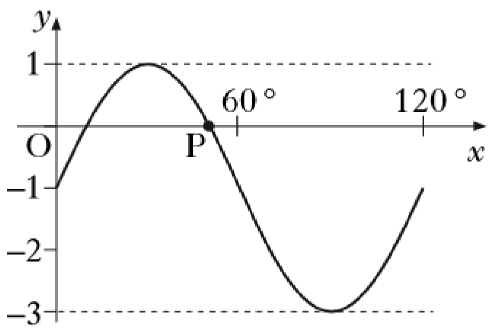
Solve the equation $\sin x° - \sin 2x° = 0$ in the interval $0 \leq x \leq 360$. **4**

8)

Solve the equation $\sin 2x° = 6\cos x°$ for $0 \leq x \leq 360$. **4**

9)

The diagram shows part of the graph of a function whose equation is of the form $y = a\sin(bx°) + c$.



(*a*) Write down the values of a, b and c. **3**

(*b*) Determine the exact value of the x-coordinate of P, the point where the graph intersects the x-axis as shown in the diagram. **3**

Differentiating Functions

1)

A curve has equation $y = x - \frac{16}{\sqrt{x}}, \quad x > 0.$

Find the equation of the tangent at the point where $x = 4$. **6**

2)

Find the equation of the tangent to the curve $y = 2\sin\left(x - \frac{\pi}{6}\right)$ at the point where $x = \frac{\pi}{3}$. **4**

3)

Given that $f(x) = \sqrt{x} + \frac{2}{x^2}$, find $f'(4)$. **5**

4)

If $f(x) = \cos(2x) - 3\sin(4x)$, find the exact value of $f'\left(\frac{\pi}{6}\right)$. **4**

5)

Given that $y = 3\sin(x) + \cos(2x)$, find $\frac{dy}{dx}$. **3**

6)

The point $P(x, y)$ lies on the curve with equation $y = 6x^2 - x^3$.

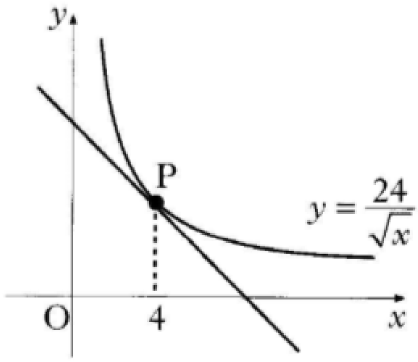
(a) Find the value of x for which the gradient of the tangent at P is 12. **5**

(b) Hence find the equation of the tangent at P. **2**

7)

The diagram shows the graph of $y = \frac{24}{\sqrt{x}}$, $x > 0$.

Find the equation of the tangent at P, where $x = 4$. **6**



8)

A function f is defined by $f(x) = (2x - 1)^5$.

Find the coordinates of the stationary point on the graph with equation $y = f(x)$ and determine its nature. **7**

9)

If $y = \frac{1}{x^3} - \cos 2x$, $x \neq 0$, find $\frac{dy}{dx}$. **4**

10)

A function f is defined by the formula $f(x) = 3x - x^3$.

(*a*) Find the exact values where the graph of $y = f(x)$ meets the x- and y-axes. **2**

(*b*) Find the coordinates of the stationary points of the function and determine their nature. **7**

(*c*) Sketch the graph of $y = f(x)$. **1**

11)

Given that $y = \sqrt{3x^2 + 2}$, find $\frac{dy}{dx}$. **3**

Integrating Functions

1)

Find $\int \frac{(x^2-2)(x^2+2)}{x^2}dx, \quad x \neq 0$ **4**

2)

A curve for which $\frac{dy}{dx} = 3\sin(2x)$ passes through the point $\left(\frac{5}{12}\pi, \sqrt{3}\right)$.

Find y in terms of x. **4**

3)

A point moves in a straight line such that its acceleration a is given by $a = 2(4-t)^{\frac{1}{2}}$, $0 \leq t \leq 4$. If it starts at rest, find an expression for the velocity v where $a = \frac{dv}{dt}$. **4**

4)

Find $\int \frac{4x^3-1}{x^2}\, dx, \quad x \neq 0.$ **4**

5)

Find $\int_0^1 \frac{dx}{(3x+1)^{\frac{1}{2}}}$. **4**

6)

Find $\int_0^2 \sqrt{4x+1}\, dx.$ **5**

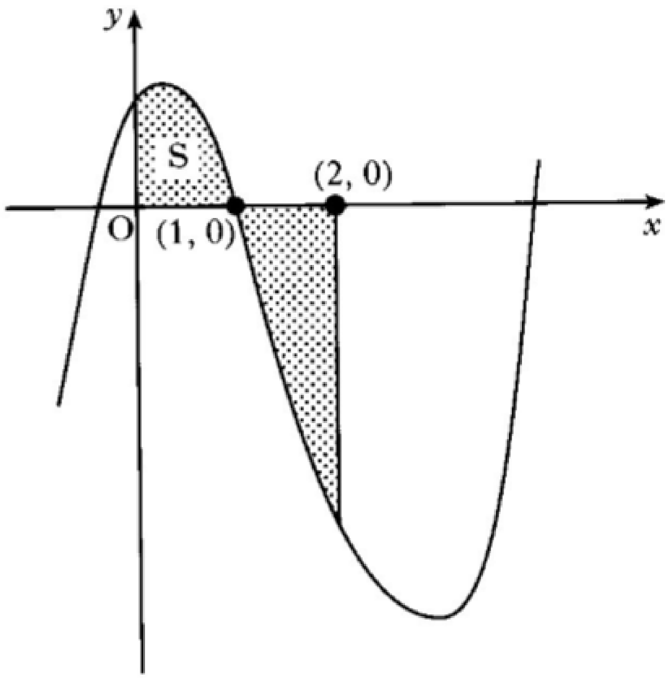
7)

The graph shown has equation $y = x^3 - 6x^2 + 4x + 1$.

The total shaded area is bounded by the curve, the x-axis, the y-axis and the line $x = 2$.

(*a*) Calculate the shaded area labelled S. **4**

(*b*) Hence find the total shaded area. **3**



8)

The curve $y = f(x)$ is such that $\frac{dy}{dx} = 4x - 6x^2$. The curve passes through the point $(-1, 9)$. Express y in terms of x. **4**

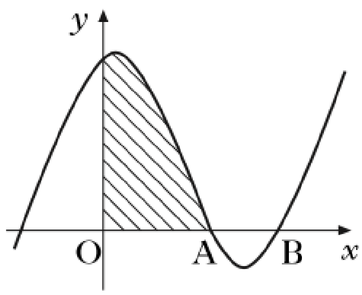
9)

The diagram shows a sketch of the graph of $y = x^3 - 4x^2 + x + 6$.

(*a*) Show that the graph cuts the x-axis at (3, 0). **1**

(*b*) Hence or otherwise find the coordinates of A. **3**

(*c*) Find the shaded area. **5**



10)

Find the value of $\int_0^2 \sin(4x + 1)\,dx$. **4**