## **Mathematics**

# Higher

Assessment 1

### **Revision Materials**

### Vectors and Straight line Skill Builder

Layout and content of the Unit Assessment will be different. This is not meant to be a carbon copy of the Unit Assessment. This booklet is an opportunity to practice all of the essential skills required to pass the Unit Assessment.

This booklet should be used to identify any areas for improvement **before** you sit the Unit assessment for the first time.

Unit	Assessment standard	Description
H4LC 76 Expressions and Functions	EF1.4 Applying geometric skills to vectors	<ul> <li>The sub-skills in the Assessment Standard are:</li> <li>determining the resultant of vector pathways in three dimensions</li> <li>working with collinearity</li> <li>determining the coordinates of an internal division point of a line</li> <li>evaluating a scalar product given suitable information and determining the angle between two vectors</li> </ul>
	EF2.1 Interpreting a situation where mathematics can be used and identifying a valid strategy	Assessment Standard 2.1 is transferable across Units. For candidates undertaking the Course, Assessment Standard 2.1 should be achieved on <b>at least two</b> occasions from across the Course.
H22J 76 Applications	APP1.1 Applying algebraic skills to rectilinear shapes	<ul> <li>The sub-skills in the Assessment Standard are:</li> <li>finding the equation of a line parallel to, and a line perpendicular to, a given line</li> <li>using m = tan θ to calculate a gradient or angle</li> </ul>

- EF1.4 Applying geometric skills to vectors:
- Sub-skills
- Determining the resultant of vector pathways in three dimensions
- Working with collinearity
- Q1 The points *A*, *B* and *C* have coordinates (10,4,5), (1,-10,-4), (-17,m,-22) respectively.
- **a)** Write down the components of  $\overrightarrow{AB}$ .
- **b)** *A*, *B* and *C* are collinear, find the value of *m*.
- **Q2** The points A, B and C have coordinates (5,4,-3), (4,-6,-10), (1,-36,p) respectively.
- a) Write down the components of  $\overrightarrow{AB}$ .
- **b)** *A*, *B* and *C* are collinear, find the value of *p*.
- **Q3** The points *A*, *B* and *C* have coordinates (5,1,-4), (0,-7,-6), (r,-39,-14) respectively.
- a) Write down the components of  $\overrightarrow{AB}$ .
- **b)** *A*, *B* and *C* are collinear, find the value of *r*.
- Q4 The points A, B and C have coordinates (8, -10, -6), (0, 6, 4), (-24, f, 34) respectively.
- **a)** Write down the components of  $\overrightarrow{AB}$ .
- **b)** A, B and C are collinear, find the value of f.
- **Q5** The points A, B and C have coordinates (10,3,1), (-5,3,9), (g,3,41) respectively.
- **a)** Write down the components of  $\overrightarrow{AB}$ .
- **b)** A, B and C are collinear, find the value of g.
- **Q6** The points A, B and C have coordinates (t, 6, 6), (9, 4, -6), (16, 2, -18) respectively.
- **a)** Write down the components of  $\overrightarrow{AB}$ .
- **b)** *A*, *B* and *C* are collinear, find the value of *t*.
- **Q7** The points *A*, *B* and *C* have coordinates (1, -3, 10), (6, 2, k), (16, 12, -44) respectively.
- **a)** Write down the components of  $\overrightarrow{AB}$ .
- **b)** *A*, *B* and *C* are collinear, find the value of *k*.

- EF1.4 Applying geometric skills to vectors:
- Sub-skills
- Determining the resultant of vector pathways in three dimensions
- Working with collinearity
- **Q8** The points *A*, *B* and *C* have coordinates (-4,2,-4), (-4,4,-3), (-4,1,-5) respectively.
- **a)** Write down the components of  $\overrightarrow{AB}$  and  $\overrightarrow{BC}$ .
- **b)** Are *A*, *B* and *C* collinear?
- **Q9** The points *A*, *B* and *C* have coordinates (4, -5, 2), (-2, 5, 1), (-20, 35, -2) respectively.
- **a)** Write down the components of  $\overrightarrow{AB}$  and  $\overrightarrow{BC}$ .
- **b)** Are *A*, *B* and *C* collinear?
- **Q10** The points *A*, *B* and *C* have coordinates (-5,2,1), (-8,-6,10), (-17,-30,37) respectively.
- **a)** Write down the components of  $\overrightarrow{AB}$  and  $\overrightarrow{BC}$ .
- **b)** Are *A*, *B* and *C* collinear?
- **Q11** The points *A*, *B* and *C* have coordinates (3, -4,1), (9,8,10), (27,44,37) respectively.
- **a)** Write down the components of  $\overrightarrow{AB}$  and  $\overrightarrow{BC}$ .
- **b)** Are *A*, *B* and *C* collinear?
- **Q12** The points A, B and C have coordinates (-2, -2, 3), (2, 4, 1), (6, 8, -1) respectively.
- **a)** Write down the components of  $\overrightarrow{AB}$  and  $\overrightarrow{BC}$ .
- **b)** Are *A*, *B* and *C* collinear?
- **Q13** The points *A*, *B* and *C* have coordinates (1, -5, -4), (-4, 0, -2), (-14, 10, 2) respectively.
- **a)** Write down the components of  $\overrightarrow{AB}$  and  $\overrightarrow{BC}$ .
- **b)** Are *A*, *B* and *C* collinear?
- Q14 The points A, B and C have coordinates (3, -5, 4), (-4, 3, -5), (5, -5, 4) respectively.
- **a)** Write down the components of  $\overrightarrow{AB}$  and  $\overrightarrow{BC}$ .
- **b)** Are *A*, *B* and *C* collinear?

- EF1.4 Applying geometric skills to vectors:
- Sub-skills
- Determining the resultant of vector pathways in three dimensions
- Determining the coordinates of an internal division point of a line
- **Q15** Point *B* divides  $\overrightarrow{AC}$  in a ratio of 1:3, where *A* and *C* are (2, -5, 0), (18, -5, -4) respectively. Find the coordinates of *B*.
- **Q16** Point *B* divides  $\overrightarrow{AC}$  in a ratio of 2:1, where *A* and *B* are (-5,2,1), (-11, -1,16) respectively. Find the coordinates of *B*.
- **Q17** Point *B* divides  $\overrightarrow{AC}$  in a ratio of 2:3, where *A* and *C* are (3,2,2), (23,17,27) respectively. Find the coordinates of *B*.
- Q18 Point *B* divides AC in a ratio of 3:2, where *A* and *C* are (-1, -1, -4), (9, -11, -9) respectively.
  Find the coordinates of *B*.
- **Q19** Point *B* divides  $\overrightarrow{AC}$  in a ratio of 3:2, where *A* and *B* are (4,1, -4), (-11, -2,8) respectively. Find the coordinates of *C*.
- **Q20** Point *B* divides  $\overrightarrow{AC}$  in a ratio of 3:1, where *B* and *C* are (-5,3,3), (-8,3,4) respectively. Find the coordinates of *A*.
- **Q21** Point *B* divides  $\overrightarrow{AC}$  in a ratio of 2:3, where *A* and *B* are (5, -5, 4), (9, -3, -2) respectively. Find the coordinates of *C*.
- **Q22** Point *B* divides  $\overrightarrow{AC}$  in a ratio of 1:2, where *A* and *C* are (-5,0,-4), (7,9,-7) respectively. Find the coordinates of *B*.
- **Q23** Point *B* divides  $\overrightarrow{AC}$  in a ratio of 2:1, where *B* and *C* are (11,2,2), (15,5,2) respectively. Find the coordinates of *A*.

- EF1.4 Applying geometric skills to vectors:
- Sub-skills
- Determining the resultant of vector pathways in three dimensions
- Determining the coordinates of an internal division point of a line
- **Q24** The points *A*, *B* and *C* have coordinates (-5, -5, 3), (-10, -1, 5), (-25, 11, 11) respectively. Find the ratio in which *B* divides  $\overrightarrow{AC}$ .
- **Q25** The points *A*, *B* and *C* have coordinates (-3,2,-3), (-3,17,12), (-3,22,17) respectively. Find the ratio in which *B* divides  $\overrightarrow{AC}$ .
- **Q26** The points *A*, *B* and *C* have coordinates (4,4,3), (4,4,8), (4,4,18) respectively. Find the ratio in which *B* divides  $\overrightarrow{AC}$ .
- **Q27** The points *A*, *B* and *C* have coordinates (-1, -2, -5), (5,0,3), (8,1,7) respectively. Find the ratio in which *B* divides  $\overrightarrow{AC}$ .
- **Q28** The points *A*, *B* and *C* have coordinates (-5,5,5), (-5,6,1), (-5,9,-11) respectively. Find the ratio in which *B* divides  $\overrightarrow{AC}$ .
- **Q29** The points *A*, *B* and *C* have coordinates (4, -3, -4), (-2, 5, -14), (-5, 9, -19) respectively. Find the ratio in which *B* divides  $\overrightarrow{AC}$ .
- **Q30** The points *A*, *B* and *C* have coordinates (-1, -4, 4), (-2, -7, 6), (-4, 13, 10) respectively. Find the ratio in which *B* divides  $\overrightarrow{AC}$ .
- **Q31** The points *A*, *B* and *C* have coordinates (3,3,-2), (5,-5,8), (6,-9,13) respectively. Find the ratio in which *B* divides  $\overrightarrow{AC}$ .
- **Q32** The points *A*, *B* and *C* have coordinates (-3, -5, -1), (-9, -17, 5), (-13, -25, 9) respectively. Find the ratio in which *B* divides  $\overrightarrow{AC}$ .

- EF1.4 Applying geometric skills to vectors:
- Sub-skills
- Evaluating a scalar product given suitable information and determining the angle between two vectors

**Q33** 
$$\overrightarrow{AB} = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}, \quad \overrightarrow{AC} = \begin{pmatrix} -5 \\ -1 \\ 2 \end{pmatrix}$$

Calculate 
$$\angle BAC$$
.

**Q34** 
$$\overrightarrow{AB} = \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}, \quad \overrightarrow{AC} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$$

**Q35** 
$$\overrightarrow{AB} = \begin{pmatrix} -5\\2\\-5 \end{pmatrix}, \quad \overrightarrow{AC} = \begin{pmatrix} -4\\-2\\0 \end{pmatrix}$$

Calculate  $\angle BAC$ .

**Q36** 
$$\overrightarrow{AB} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix}, \quad \overrightarrow{AC} = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$$

Calculate 
$$\angle BAC$$
.

**Q37**  $\overrightarrow{AB} = \begin{pmatrix} 3 \\ -3 \\ 2 \end{pmatrix}, \quad \overrightarrow{AC} = \begin{pmatrix} 2 \\ -5 \\ 4 \end{pmatrix}$ 

Calculate 
$$\angle BAC$$
.

**Q38** 
$$\overrightarrow{AB} = \begin{pmatrix} 0\\0\\4 \end{pmatrix}, \quad \overrightarrow{AC} = \begin{pmatrix} 3\\5\\4 \end{pmatrix}$$

Calculate 
$$\angle BAC$$
.

**Q39** 
$$\overrightarrow{AB} = \begin{pmatrix} -5\\1\\4 \end{pmatrix}, \quad \overrightarrow{AC} = \begin{pmatrix} -4\\-5\\5 \end{pmatrix}$$

Calculate  $\angle BAC$ .

**Q40** 
$$\overrightarrow{AB} = \begin{pmatrix} -1\\0\\1 \end{pmatrix}, \quad \overrightarrow{AC} = \begin{pmatrix} 2\\5\\-5 \end{pmatrix}$$

Calculate  $\angle BAC$ .

EF1.4 Applying geometric skills to vectors:

- Sub-skills
- Determining the resultant of vector pathways in three dimensions

Q41 *ABCD*, *EFGH* is a cuboid.  

$$\overrightarrow{AB} = \begin{pmatrix} 3\\1\\2 \end{pmatrix}$$
,  $\overrightarrow{CB} = \begin{pmatrix} 3\\1\\-5 \end{pmatrix}$  and  $\overrightarrow{FB} = \begin{pmatrix} -1\\3\\0 \end{pmatrix}$   
Write down1 the components of :  
a)  $\overrightarrow{AH}$  b)  $\overrightarrow{AG}$  c)  $\overrightarrow{CE}$  d)  $\overrightarrow{BH}$ 

Q42 ABCD, EFGH is a cuboid.  

$$\overrightarrow{AB} = \begin{pmatrix} -3 \\ -2 \\ 2 \end{pmatrix}, \ \overrightarrow{CB} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} and \ \overrightarrow{DH} = \begin{pmatrix} -2 \\ 10 \\ 7 \end{pmatrix}$$
  
Write down the components of :  
a)  $\overrightarrow{AH}$  b)  $\overrightarrow{GA}$  c)  $\overrightarrow{CE}$  d)  $\overrightarrow{HB}$ 

c)

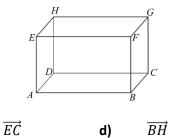
c)

**Q43** *ABCD*, *EFGH* is a cuboid.

$$\overrightarrow{BA} = \begin{pmatrix} -1\\0\\4 \end{pmatrix}, \ \overrightarrow{BC} = \begin{pmatrix} 8\\5\\2 \end{pmatrix} \ and \ \overrightarrow{FB} = \begin{pmatrix} -20\\34\\5 \end{pmatrix}$$

Write down the components of :

a) 
$$\overrightarrow{AH}$$
 b)  $\overrightarrow{AG}$ 



**Q44** *ABCD*, *EFGH* is a cuboid.

$$\overrightarrow{AB} = \begin{pmatrix} -3\\1\\1 \end{pmatrix}$$
,  $\overrightarrow{CB} = \begin{pmatrix} 2\\1\\5 \end{pmatrix}$  and  $\overrightarrow{BF} = \begin{pmatrix} 4\\17\\-5 \end{pmatrix}$ 

H G E F C A B



Write down the components of :

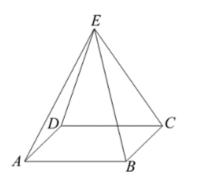


- EF1.4 Applying geometric skills to vectors:
- Sub-skills

Q47

- Determining the resultant of vector pathways in three dimensions
- **Q45** *ABCD*, *EFGH* is a rectangular based pyramid.

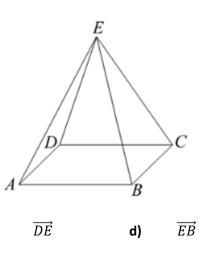
$$\overrightarrow{AB} = \begin{pmatrix} 4\\2\\4 \end{pmatrix}$$
,  $\overrightarrow{CB} = \begin{pmatrix} 4\\-4\\-2 \end{pmatrix}$  and  $\overrightarrow{EC} = \begin{pmatrix} 8\\1\\-7 \end{pmatrix}$ 



Write down the components of :



Q46 ABCD, EFGH is a rectangular based pyramid.  $\overrightarrow{BA} = \begin{pmatrix} -4 \\ -6 \\ -2 \end{pmatrix}, \ \overrightarrow{CB} = \begin{pmatrix} 4 \\ -2 \\ -2 \end{pmatrix} and \ \overrightarrow{EC} = \begin{pmatrix} -8 \\ 6 \\ 16 \end{pmatrix}$ Write down the components of : a)  $\overrightarrow{CA}$  b)  $\overrightarrow{AE}$  c)  $\overrightarrow{DE}$  d)  $\overrightarrow{BE}$ 



c)

Write down the components of :

a)  $\overrightarrow{AC}$  b)  $\overrightarrow{EA}$ 

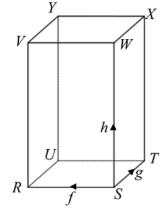
ABCD, EFGH is a rectangular based pyramid.

 $\overrightarrow{AB} = \begin{pmatrix} 4\\4\\8 \end{pmatrix}, \ \overrightarrow{CB} = \begin{pmatrix} -4\\-4\\4 \end{pmatrix} \ and \ \overrightarrow{EC} = \begin{pmatrix} 4\\-4\\-6 \end{pmatrix}$ 

#### **MIXED QUESTIONS**

- EF1.4 Applying geometric skills to vectors:
- Sub-skills
- Determining the resultant of vector pathways in three dimensions
- Working with collinearity
- Determining the coordinates of an internal division point of a line
- Evaluating a scalar product given suitable information and determining the angle between two vectors

**Q48** In the diagram RSTU, VWXY represents a cuboid.  $\overline{SR}$  represents vector f,  $\overline{ST}$  represents vector gand  $\overline{SW}$  represents vector h. Express  $\overline{VT}$  in terms of f, g and h.

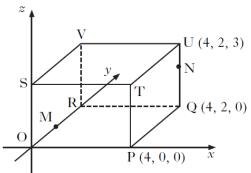


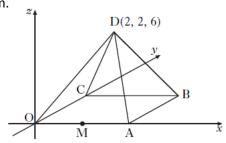
**Q49** The diagram shows a cuboid OPQR, STUV relative to the coordinate axes. *P* is the point (4,0,0), *Q* is (4,2,0) and *U* is (4,2,3).

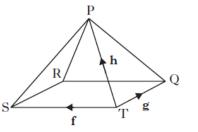
M is the midpoint of OR.

N is the point on UQ such that  $UN = \frac{1}{2}UQ$ .

- a) State the coordinates of *M* and *N*.
- **b)** Express  $\overrightarrow{VM}$  and  $\overrightarrow{VN}$  in component form.
- c) Calculate the size of  $\angle MVN$ .







- **Q50** D, OABC is a square based pyramid as shown in the diagram. O is the origin, D is the point (2, 2, 6), M is the mid point of  $\overrightarrow{OA}$  and OA = 4 units.
  - a) State the coordinates of *B*.
  - **b)** Express  $\overrightarrow{DB}$  and  $\overrightarrow{DM}$  in component form.
  - c) Find the size of  $\angle BDM$ .

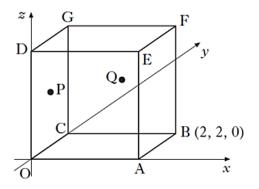
**Q51** The diagram shows a square based pyramid P, QRST.  $\overrightarrow{TS}, \overrightarrow{TQ}$  and  $\overrightarrow{TP}$  represents f, g and h respectively. Express  $\overrightarrow{RP}$  in terms of f, g and h.

#### **Q52** *OABCDEFG* is a cube with sides 2 *units* as shown in the diagram.

B has the coordinates (2, 2, 0).

*P* is the centre of face *OCGD*.

Q is the centre of Face CBFG.



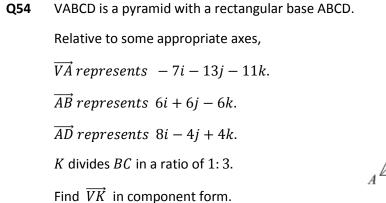
- a) Write down the coordinates of *G*.
- **b)** Find p and q, the position vectors of the points P and Q.
- c) Find the size of  $\angle POQ$ .

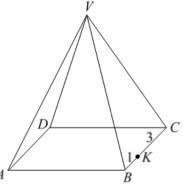
Q53 The diagram shows a cuboid OABC, DEFG.
F is the point (8, 4, 6).
P divides AE in the ratio 2: 1.
Q is the midpoint of CG.



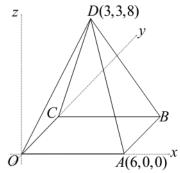
**b)** Write down the components of *PQ* and *PA*.

c) Find the size of  $\angle QPA$ .

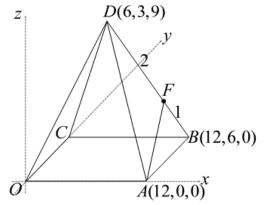




- Q55 The diagram shows a square based pyramid of height 8 *units*. Square OABC has a side length of 6 *units*. *z* The coordinates of *A* and *D* are (6,0,0) and (3,3,8). *C* lies on the *y* axis.
  a) Write down the coordinates of *B*.
  - **b)** Determine the components of  $\overrightarrow{DA}$  and  $\overrightarrow{DB}$ .
  - c) Calculate the size of  $\angle ADB$ .

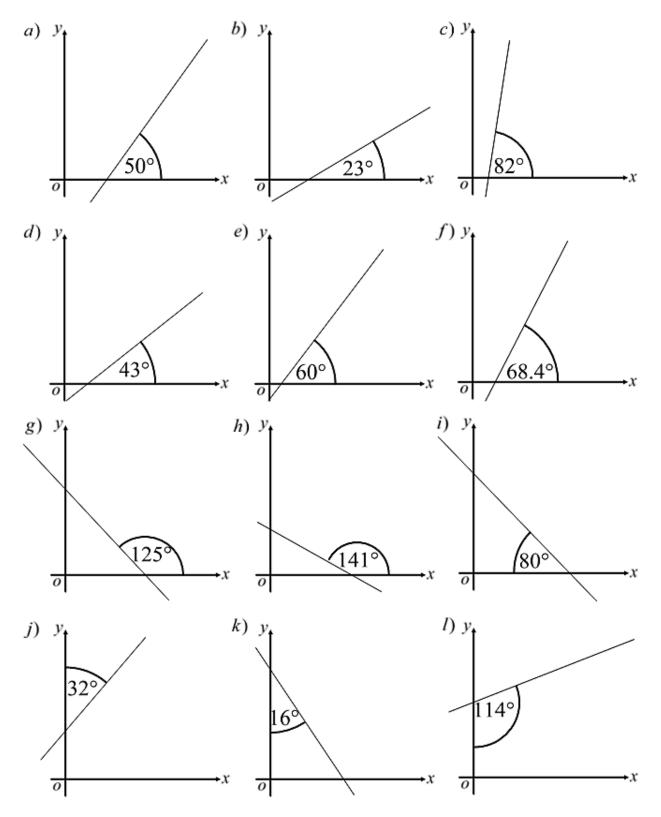


- Q56D, OABC is a pyramid.A is the point (12, 0, 0).B is the point (12, 6, 0).D is the point (6, 3, 9).F divides BD in a ratio of 1: 2.a)Find the coordinates of the point F.
  - **b)** Express  $\overrightarrow{AF}$  in component form.



- APP1.1 Applying algebraic skills to rectilinear shapes
- Sub-skills
- Using  $m = \tan \theta$  to calculate a gradient or angle

**Q57** Calculate the gradient of each of the following lines to 3 significant figures.



- APP1.1 Applying algebraic skills to rectilinear shapes
- Sub-skills
- Using  $m = \tan \theta$  to calculate a gradient or angle
- **Q58** For each of the following, calculate the angle the line makes with the positive direction of the *x axis*.

a) 
$$y = x - 3$$
 b)  $y = 2x + 1$  c)  $y = 5 - x$ 

- d) y = -2x + 1 e) 2y x + 1 = 0 f) 3y + 2 = x
- g) 5x 2y = 7 h) 2 2y x = 0 i)  $\frac{1}{3}y = \frac{1}{4}x + 1$

j) 
$$2(y-2) = 3(x+1)$$
 k)  $(2-3y) = 4x$  l)  $-2x = \frac{3}{4}y + 1$ 

- APP1.1 Applying algebraic skills to rectilinear shapes
- Sub-skills
- Finding the equation of a line parallel to, and a line perpendicular to, a given line
- **Q59** a) A line *L*, has the equation 2x + y 3 = 0. Write down the equation of a line, passing through (-1,3) which is:
  - *i*) Parallel to *L*.
  - *ii)* Perpendicular to *L*.

**b)** A line *L*, has the equation y - 3x + 7 = 0. Write down the equation of a line, passing through (1,1) which is:

- *i*) Parallel to *L*.
- *ii)* Perpendicular to *L*.

c) A line *L*, has the equation 2x - 3y - 5 = 0. Write down the equation of a line, passing through (-3,2) which is:

- *i*) Parallel to *L*.
- *ii)* Perpendicular to *L*.
- d) A line *L*, has the equation 6x + 4y + 2 = 0. Write down the equation of a line, passing through (0, -2) which is:
- *i)* Parallel to *L*.
- *ii)* Perpendicular to *L*.
- e) A line L, has the equation 7y 4x = 0. Write down the equation of a line, passing through (-4, -2) which is:
- *i)* Parallel to *L*.
- *ii)* Perpendicular to *L*.
- f) A line L, has the equation  $\frac{1}{2}x + \frac{1}{3}y 1 = 0$ . Write down the equation of a line, passing through (-4,0) which is:
- *i*) Parallel to *L*.
- *ii)* Perpendicular to *L*.

g) A line L, has the equation 
$$\frac{2}{3}y + x - 3 = 0$$
.  
Write down the equation of a line, passing through (2,2) which is:

- *i*) Parallel to *L*.
- *ii)* Perpendicular to *L*.

**h)** A line *L*, has the equation 
$$\frac{x+y-4}{2} = 0$$
.

Write down the equation of a line, passing through (0, -7) which is:

- *i)* Parallel to *L*.
- *ii)* Perpendicular to *L*.

Answers.

Q1	a)	$\begin{pmatrix} -9\\ -14\\ -9 \end{pmatrix}$		b)	m = -1	38		
Q2	a)	$\begin{pmatrix} -1 \\ -10 \\ -7 \end{pmatrix}$		b)	<i>p</i> = 31			
Q3	a)	$\begin{pmatrix} -5\\ -8\\ -2 \end{pmatrix}$		b)	r = -2	20		
Q4	a)	$\begin{pmatrix} -8\\16\\10 \end{pmatrix}$		b)	<i>f</i> = 54			
Q5	a)	$\begin{pmatrix} -15\\0\\8 \end{pmatrix}$		b)	g = -7	75		
Q6	a)	$ \begin{pmatrix} 9-t \\ -2 \\ -12 \end{pmatrix} $	)	b)	<i>t</i> = 2			
Q7	a)	$\begin{pmatrix} 5\\5\\k-1 \end{pmatrix}$		b)	k = -8	3		
Q8	$\overrightarrow{AB} =$	$\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$	$\overrightarrow{BC} = ($	$\begin{pmatrix} 0\\ -3\\ -2 \end{pmatrix}$		Not collinear		
Q9	$\overrightarrow{AB} =$	$\begin{pmatrix} -6\\10\\1 \end{pmatrix}$	$\overrightarrow{BC} = 3$	$3\begin{pmatrix} -6\\10\\-1 \end{pmatrix}$		Collinear		
Q10	$\overrightarrow{AB} =$	$\begin{pmatrix} -3\\ -8\\ 9 \end{pmatrix}$	$\overrightarrow{BC} = 3$	$3\begin{pmatrix} -3\\ -8\\ 9 \end{pmatrix}$		Collinear		
Q11	$\overrightarrow{AB} =$	$3\begin{pmatrix}2\\4\\3\end{pmatrix}$	$\overrightarrow{BC} = 9$	$ \Theta\begin{pmatrix}2\\4\\3\end{pmatrix} $		Collinear		
Q12	$\overrightarrow{AB} =$	$2\begin{pmatrix}2\\3\\-1\end{pmatrix}$	$\overrightarrow{BC} = 2$	$2\begin{pmatrix}2\\2\\-1\end{pmatrix}$		Not collinear		
Q13	$\overrightarrow{AB} =$	$\binom{-5}{5}{2}$	$\overrightarrow{BC} = 2$	$2\binom{-5}{5}{2}$		Collinear		
Q14	$\overrightarrow{AB} =$	$\begin{pmatrix} -7\\8\\-9 \end{pmatrix}$	$\overrightarrow{BC} =$	$\begin{pmatrix} 9\\-8\\9 \end{pmatrix}$		Not collinear		
Q15	B(6,-	-5, -1)		Q16	B(-9,0	0,11)	Q17	B(11, 8, 12)
Q18	B(5,-	-7, -7)		Q19	C(-21	, -4, 16)	Q20	A(1, 3, 1)
Q21	С(15,	0,-11)		Q22	B(-1,3	3, -5)	Q23	A(3, -4, 2)

b) 
$$\overrightarrow{VM} = \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix} \quad \overrightarrow{VN} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$
  
c)  $\angle MVN = 76.697 \dots^{\circ}$ 

$$\Delta M V N = 76.$$

b) 
$$\overrightarrow{DB} = \begin{pmatrix} 2\\ 2\\ -6 \end{pmatrix} \qquad \overrightarrow{DM} = \begin{pmatrix} 0\\ -2\\ -6 \end{pmatrix}$$

c) 
$$\angle BDM = 40.290 \dots^{\circ}$$

Q51 
$$\overrightarrow{RP} = -f - g + h$$

Q52 a) 
$$G(0, 2, 2)$$
  
b)  $p = \begin{pmatrix} 0\\1\\1 \end{pmatrix} q = \begin{pmatrix} 1\\2\\1 \end{pmatrix}$   
c)  $\angle POQ = 30^{\circ}$ 

Q53 a) 
$$P(8, 0, 4)$$
  $Q(0, 4, 3)$   
b)  $\overrightarrow{PQ} = \begin{pmatrix} -8\\4\\-1 \end{pmatrix}$   $\overrightarrow{PA} = \begin{pmatrix} 0\\0\\-4 \end{pmatrix}$   
c)  $\angle QPA = 83.620 \dots^{\circ}$ 

Q54 
$$\overrightarrow{VK} = \begin{pmatrix} 1\\ -8\\ -16 \end{pmatrix}$$

Q55 a) 
$$B(6, 6, 0)$$
  
b)  $\overrightarrow{DA} = \begin{pmatrix} 3 \\ -3 \\ -8 \end{pmatrix} \qquad \overrightarrow{DB} = \begin{pmatrix} 3 \\ 3 \\ -8 \end{pmatrix}$   
c)  $\angle ADB = 38.964 \dots^{\circ}$ 

Q56 a) 
$$F(10, 5, 3)$$
  
b)  $\overrightarrow{AF} = \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix}$ 

Q57	a)	1.191	b)	0.424	c)	7.115
	d)	0.932	e)	$\sqrt{3} \ or \ 1.732 \dots$	f)	2.525
	g)	-1.428	h)	-0.809	i)	-5.671
	j)	1.600	k)	-3.487	I)	0.445
Q58	a)	45 °	b)	63.434 °	c)	135 °
	d)	116.56 °	e)	26.565 °	f)	18.434 °
	g)	68.198 °	h)	153.434 °	i)	36.869 °
	j)	56.309 °	k)	126.869 °	I)	110.556 °

Q59 a) i) 
$$y - 3 = -2(x + 1)$$
  
ii)  $y - 3 = \frac{1}{2}(x + 1)$   
b) i)  $y - 1 = 3(x - 1)$   
ii)  $y - 1 = -\frac{1}{3}(x - 1)$   
c) i)  $y - 2 = \frac{2}{3}(x + 3)$   
ii)  $y - 2 = -\frac{3}{2}(x + 3)$   
d) i)  $y + 2 = -\frac{3}{2}x$   
ii)  $y + 2 = \frac{2}{3}x$   
e) i)  $y + 2 = \frac{2}{3}x$   
e) i)  $y + 2 = \frac{4}{7}(x + 4)$   
ii)  $y + 2 = -\frac{7}{4}(x + 4)$   
f) i)  $y = -\frac{3}{2}(x + 4)$   
ii)  $y = \frac{2}{3}(x + 4)$   
g) i)  $y - 2 = -\frac{3}{2}(x - 2)$   
ii)  $y - 2 = \frac{2}{3}(x - 2)$   
h) i)  $y + 7 = -x$ 

ii) y + 7 = x