Mathematics Higher

Assessment 4

Revision Materials

Trigonometry Skill Builder

Layout and content of the Unit Assessment will be different. This is not meant to be a carbon copy of the Unit Assessment. This booklet is an opportunity to practice all of the essential skills required to pass the Unit Assessment.

This booklet should be used to identify any areas for improvement **before** you sit the Unit assessment for the first time.

Unit	Assessment standard	Description
H4LC 76 Expressions and Functions	1.2 Applying trigonometric skills to manipulating expressions	The sub-skills in the Assessment Standard are: ♦ applying the addition or double angle formulae ♦ applying trigonometric identities ♦ converting $a\cos x + b\sin x$ to $k\cos(x \pm \alpha)$ or $\sin(x \pm \alpha)$, for α in $1st$ $Quadrant$ and $k > 0$.
	EF#2.2 Explaining a solution and, where appropriate, relating it to context	Assessment Standard 2.2 is transferable across Units. For candidates undertaking the Course, Assessment Standard 2.2 should be achieved on **at least two** occasions from across the Course.
H4LD 76 Relationships and Calculus	1.2 Applying trigonometric skills to solve equations	The sub-skills in the Assessment Standard are: ♦ solving trigonometric equations in degrees, including those involving trigonometric formulae or identities, in a given interval
	AP#2.1 Explaining a solution and, where appropriate, relating it to context.	Assessment Standard 2.1 is transferable across Units. For candidates undertaking the Course, Assessment Standard 2.1 should be achieved on **at least two** occasions from across the Course.

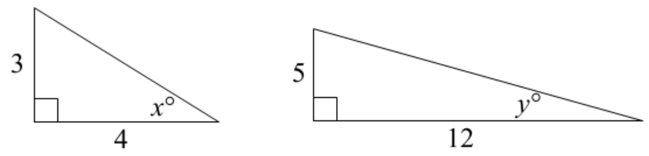
Expressions and Functions

1.2 Applying trigonometric skills to manipulating expressions.

Sub-skills

- **applying the addition or double angle formulae**

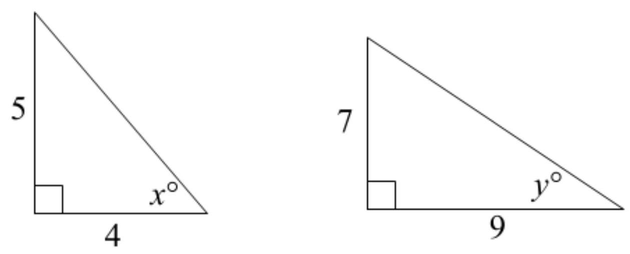
Q1. The diagram below shows two right angled triangles.



Use the triangles to find the exact values for:

a)	$\sin(x+y)$	**b)**	$\sin(x-y)$	**c)**	$\cos(x+y)$	**d)**	$\cos(x-y)$
e)	$\sin(2x)$	**f)**	$\sin(2y)$	**g)**	$\cos(2x)$	**h)**	$\cos(2y)$
i)	$\tan x$	**j)**	$\tan(x+y)$	**k)**	$\tan(x-y)$	**l)**	$\tan 2x$

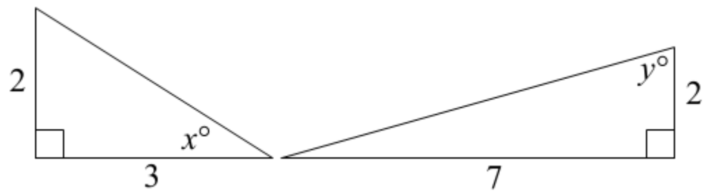
Q2. The diagram below shows two right angled triangles.



Use the triangles to find the exact values for:

a)	$\sin(x+y)$	**b)**	$\sin(x-y)$	**c)**	$\cos(x+y)$	**d)**	$\cos(x-y)$
e)	$\sin(2x)$	**f)**	$\sin(2y)$	**g)**	$\cos(2x)$	**h)**	$\cos(2y)$
i)	$\tan x$	**j)**	$\tan(x+y)$	**k)**	$\tan(x-y)$	**l)**	$\tan 2x$

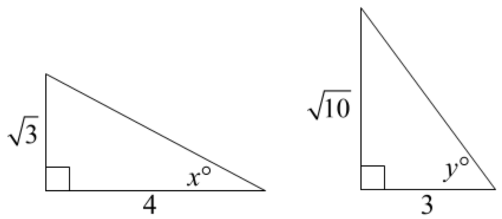
Q3. The diagram below shows two right angled triangles.



Use the triangles to find the exact values for:

a)	$\sin(x+y)$	**b)**	$\sin(x-y)$	**c)**	$\cos(x+y)$	**d)**	$\cos(x-y)$
e)	$\sin(2x)$	**f)**	$\sin(2y)$	**g)**	$\cos(2x)$	**h)**	$\cos(2y)$
i)	$\tan x$	**j)**	$\tan(x+y)$	**k)**	$\tan(x-y)$	**l)**	$\tan 2x$

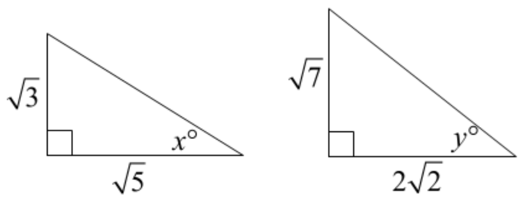
Q4. The diagram below shows two right angled triangles.



Use the triangles to find the exact values for:

a)	$\sin(x+y)$	**b)**	$\sin(x-y)$	**c)**	$\cos(x+y)$	**d)**	$\cos(x-y)$
e)	$\sin(2x)$	**f)**	$\sin(2y)$	**g)**	$\cos(2x)$	**h)**	$\cos(2y)$
i)	$\tan x$	**j)**	$\tan(x+y)$	**k)**	$\tan(x-y)$	**l)**	$\tan 2x$

Q5. The diagram below shows two right angled triangles.



Use the triangles to find the exact values for:

a)	$\sin(x+y)$	**b)**	$\sin(x-y)$	**c)**	$\cos(x+y)$	**d)**	$\cos(x-y)$
e)	$\sin(2x)$	**f)**	$\sin(2y)$	**g)**	$\cos(2x)$	**h)**	$\cos(2y)$
i)	$\tan x$	**j)**	$\tan(x+y)$	**k)**	$\tan(x-y)$	**l)**	$\tan 2x$

Expressions and Functions

1.2 Applying trigonometric skills to manipulating expressions.

Sub-skills

- **converting** $a\cos x + b\sin x$ **to** $k\cos(x \pm \alpha)$ *or* $k\sin(x \pm \alpha)$, *for* α *in* 1*st Quadrant and* $k > 0$.

Q6. Express each of the following in the form $k\cos(x - \alpha)$, $0 \le x \le 2\pi$

a) $\cos x + \sin x$ **b)** $\cos x - \sqrt{3}\sin x$

c) $\sqrt{3}\cos x + \sin x$ **d)** $2\cos x + 2\sin x$

Q7. Express each of the following in the form $k\sin(x - \alpha)$, $0 \le x \le 2\pi$

a) $3\cos x - 4\sin x$ **b)** $8\cos x + 6\sin x$

c) $-\cos x + 2\sin x$ **d)** $-\cos x - 3\sin x$

Q8. Express $3\cos x + 4\sin x$ in each of the following forms: $0° \le x \le 360°$

a) $k\cos(x - \alpha)°$ **b)** $k\cos(x + \alpha)°$

c) $k\sin(x - \alpha)°$ **d)** $k\sin(x + \alpha)°$

Q9. Express each of these in the form $k\cos(bx - \alpha)$: $0 \le x \le 2\pi$

a) $\cos 2x + \sin 2x$ **b)** $sin\ 3x - \cos 3x$

Q10. Express each of the following in the given form, for $0 \le x \le 2\pi$, use RAD mode on your calculator.

a) $3\cos 2\theta - 4\sin 2\theta$; $k\cos(2\theta - \alpha)$

b) $2\sin 2\theta + \cos 2\theta$; $k\sin(2\theta - \alpha)$

Q11. Express each of the following in the form $k\cos(x - \alpha)°$ for $0° \le x° \le 360°$

a) $12\cos x + 5\sin x$ **b)** $7\cos x - 24\sin x$

Q12. Express the left hand side of each equation in the form $k\cos(x - \alpha)°$

for $0° \le x° \le 360°$, then solve.

a) $\cos x + \sin x = 1$ **b)** $3\cos x + 4\sin x = 5$

c) $9\sin x - 12\cos x = 10$ **d)** $2\cos x + 3\sin x = -1$

Q13. Express the left hand side of each equation in the form $k \sin(x - \alpha)°$

$for\ 0° \leq x° \leq 360°$, then solve.

a) $3 \sin x - 4 \cos x = 2$ **b)** $4 \cos x - 3 \sin x = 5$

c) $6 \sin x - 8 \cos x = 10$ **d)** $\cos x - \sqrt{3} \sin x = 1$

Q14. Express $8 \cos x° - 6 \sin x°$ in the form $k \cos(x° + \alpha°)$ $where\ k > 0\ and\ 0 < \alpha < 360.$

Q15. Express $2 \sin x° - 5 \cos x°$ in the form $k \sin(x° - \alpha°)$ $where\ k > 0\ and\ 0 \leq \alpha \leq 360.$

Q16. **a)** Express $\sin x° - 3 \cos x°$ in the form $k \sin(x° - \alpha°)$ $where\ k > 0$ $and\ 0 \leq \alpha \leq 360.$ Find the values of $k\ and\ \alpha.$

b) Find the maximum value of $5 + \sin x° - 3 \cos x°$ and state the value of x for which this maximum occurs.

Q17. Solve the simultaneous equations $k \sin x° = 5\ and\ k \cos x° = 2$

$where\ k > 0\ and\ 0° \leq x \leq 360°.$

Q18. Solve the equation $2 \sin x° - 3 \cos x° = 2.5$ in the interval $0° \leq x < 360°$.

Q19. **a)** Show that $2 \cos(x° + 30°) - \sin x°$ can be written as $\sqrt{3} \cos x° - 2 \sin x°.$

b) Express $\sqrt{3} \cos x° - 2 \sin x°$ in the form $k \cos(x° + \alpha°)$ $where\ k > 0$ and $0° \leq x \leq 360°$ and find the values of $k\ and\ \alpha.$

c) Hence, or otherwise, solve the equation $2 \cos(x° + 30°) = \sin x° + 1,$

$0° \leq x \leq 360°$

Q20. $f(x) = 2 \cos x° + 3 \sin x°.$

a) Express $f(x)$ in the form $k \cos(x° - \alpha°)$ $where\ k > 0$ and $0° \leq x \leq 360°$

b) Hence solve algebraically $f(x) = 0.5$ for $0° \leq x < 360°$.

Expressions and Functions

1.2 Applying trigonometric skills to manipulating expressions.

Sub-skills

- applying the addition or double angle formulae
- applying trigonometric identities

Q21. Knowing that $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$,

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$, and $\sin^2 x + \cos^2 x = 1$, prove the following:

a) $\sin 2A = 2\sin A\cos A$

b) $\cos 2A = \cos^2 A - \sin^2 A$

c) $\cos 2A = 2\cos^2 A - 1$

d) $\cos 2A = 1 - 2\sin^2 A$

e) $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$

f) $\sin^2 A = \frac{1}{2}(1 - \cos 2A)$

Q22. Show that:

a) $\frac{\tan A\cos A}{\sin A} = 1$

b) $1 + \frac{1}{\tan^2 A} = \frac{1}{\sin^2 A}$

c) $1 + \tan^2 A = \frac{1}{\cos^2 A}$

d) $\tan A - \tan B = \frac{\sin(A-B)}{\cos A\cos B}$

e) $\tan 3A + \tan A = \frac{\sin 4A}{\cos 3A\cos A}$

f) $\frac{\tan^2 A - 1}{\tan^2 A + 1} = 1 - 2\cos^2 A$

g) $\frac{\tan^2 A}{\sin^2 A} - 1 = \tan^2 A$

h) $\sin A - \sin A\cos^2 A = \sin^3 A$

i) $\frac{\cos A}{1-\sin A} - \frac{\cos A}{1+\sin A} = 2\tan A$

j) $(\sin x - \cos x)^2 + (\sin x + \cos x)^2 = 2$

k) $\frac{1-\sin x}{\cos x} = \frac{\cos x}{1+\sin x}$

l) $\sin^4 x - \cos^4 x = 1 - 2\cos^2 x$

* Most of the above are much more difficult than you will be presented with in the final exam*

**Remember $\tan A = \frac{\sin A}{\cos A}$, $\tan 2A = \frac{\sin 2A}{\cos 2A}$, ... $\tan nA = \frac{\sin nA}{\cos nA}$ **

Expressions and Functions

1.2 Applying trigonometric skills to manipulating expressions.

Sub-skills

- applying the addition or double angle formulae
- applying trigonometric identities
- **EF#2.2 Explaining a solution and, where appropriate, relating it to context**

Q23.

a) **i)** Show that $(\sin x + \cos x)^2$ can be written as $1 + \sin 2x$.

ii) Hence state the maximum and minimum vales of $(2\sin x + 2\cos x)^2$

b) **i)** Show that $\sin^3 x\ \cos x + \sin x \cos^3 x$ can be written as $\frac{1}{2}\sin 2x$.

ii) Hence state the maximum and minimum vales of

$\sqrt{3}\sin^3 x\ \cos x + \sqrt{3}\sin x \cos^3 x$

c) **i)** By using $3x = 2x + x$, show that $\sin 3x$ can be written as $3\sin x - 4\sin^3 x$

ii) Hence state the maximum and minimum vales of $\frac{3}{2}\sin x - 2\sin^3 x$

d) **i)** By using $3x = 2x + x$, show that $\cos 3x$ can be written as $4\cos^3 x - 3\cos x$

ii) Hence state the maximum and minimum vales of $-12\cos^3 x + 9\cos x$

e) **i)** By using $\tan x = \frac{\sin x}{\cos x}$, show that $\frac{2\tan x}{1+\tan^2 x}$ can be written as $\sin 2x$

ii) Hence state the maximum and minimum vales of $\frac{\tan x}{2+2\tan^2 x}$

f) **i)** By using $\tan x = \frac{\sin x}{\cos x}$, show that $\frac{1-\tan^2 x}{1+\tan^2 x}$ can be written as $\cos 2x$

ii) Hence state the maximum and minimum vales of $\frac{3-3\tan^2 x}{2+2\tan^2 x}$

Relationships and Calculus

1.2 Applying trigonometric skills to manipulating expressions.

Sub-skills

- **solving trigonometric equations in degrees, including those involving trigonometric formulae or identities, in a given interval**

Q24. Solve the following equations in the given domains

a) $\sin 2x = 1$ for $0 \leq x \leq 180$

b) $\cos 3x = -1$ for $0 \leq x \leq 360$

c) $2\sin 3x = 1$ for $0 \leq x \leq 180$

d) $-\cos 4x = 0.5$ for $0 \leq x \leq 90$

e) $2\sin 2x = \sqrt{3}$ for $0 \leq x \leq 180$

f) $4\cos 2x = 3$ for $0 \leq x \leq 360$

g) $-3\sin 2x = \sqrt{7}$ for $90 \leq x \leq 270$

h) $\sqrt{10}\cos\frac{2}{3}x = 2$ for $0 \leq x \leq 540$

i) $2\sin\frac{1}{2}x = -1$ for $360 \leq x \leq 720$

j) $0.5\cos 2x = 0.25$ for $0 \leq x \leq 720$

k) $3\sin 3x = 0.5$ for $0 \leq x \leq 2\pi$

l) $3\cos 2x = 2$ for $0 \leq x \leq 2\pi$

m) $2\sin 6x = 0$ for $0 \leq x \leq \frac{\pi}{2}$

n) $3\cos(-2x) = 1$ for $-\pi \leq x \leq \pi$

o) $6\sin 2x = -1$ for $0 \leq x \leq \pi$

p) $3\cos 4x = -1$ for $0 \leq x \leq \frac{\pi}{2}$

q) $5\sin 3x = -3$ for $0 \leq x \leq 2\pi$

r) $7\cos 2x = -4$ for $0 \leq x \leq 3\pi$

Relationships and Calculus

1.2 Applying trigonometric skills to manipulating expressions.

Sub-skills

- **solving trigonometric equations in degrees, including those involving trigonometric formulae or identities, in a given interval**
- **AP#2.1 Explaining a solution and, where appropriate, relating it to context.**

Q25. Solve the following:

a) $\sin x° \cos 10° + \sin 10° \cos x° = \frac{3}{4}$ for $0° < x < 360°$

b) $\sin x° \cos 25° + \sin 25° \cos x° = 0.5$ for $0° < x < 360°$

c) $\sin x° \cos 15° - \sin 15° \cos x° = \frac{2}{3}$ for $0° < x < 360°$

d) $\sin x° \cos 71° - \sin 71° \cos x° = -\frac{1}{4}$ for $0° < x < 360°$

e) $\cos x° \cos 14° + \sin x° \sin 14° = 0.8$ for $-180° < x < 540°$

f) $\cos x° \cos 63° + \sin x° \sin 63° = -\frac{1}{6}$ for $0° < x < 720°$

g) $\cos x° \cos 22° - \sin x° \sin 22° = \frac{\sqrt{3}}{2}$ for $0° < x < 360°$

h) $\cos x° \cos 2° - \sin x° \sin 2° = -\frac{1}{\sqrt{2}}$ for $0° < x < 180°$

i) $2\sin x° \cos 45° + 2 \sin 45° \cos x° = 1.5$ for $0° < x < 360°$

j) $3\cos x° \cos 30° - 3\sin x° \sin 30° = 2$ for $0° < x < 360°$

Relationships and Calculus

1.2 Applying trigonometric skills to manipulating expressions.

Sub-skills

- **solving trigonometric equations in degrees, including those involving trigonometric formulae or identities, in a given interval**
- **AP#2.1 Explaining a solution and, where appropriate, relating it to context.**

Q26. Solve the following:

a) $\sin 2x° - \cos x° = 0$ for $0° \leq x \leq 360°$

b) $\sin 2x° - 3\sin x° = 0$ for $0° \leq x \leq 360°$

c) $\cos 2x° + \cos x° = 0$ for $0° \leq x \leq 360°$

d) $\cos 2x° + \sin x° = 0$ for $0° \leq x \leq 360°$

e) $\cos 2x° + \cos x° + 1 = 0$ for $0° \leq x \leq 360°$

f) $\cos 2x° - 7\sin x° - 4 = 0$ for $0° \leq x \leq 360°$

g) $2\cos 2x° - 3\cos x° + 1 = 0$ for $0° \leq x \leq 360°$

h) $3\cos 2x° + \sin x° - 2 = 0$ for $0° \leq x \leq 360°$

i) $5\cos 2\theta - \cos\theta + 2 = 0$ for $0 \leq x \leq 2\pi$

j) $6\cos 2\theta - 5\cos\theta + 4 = 0$ for $0 \leq x \leq 2\pi$

ANSWERS

Q1.

a) $\frac{56}{65}$	**b)** $\frac{16}{65}$	**c)** $\frac{33}{65}$	**d)** $\frac{63}{65}$
e) $\frac{24}{25}$	**f)** $\frac{120}{169}$	**g)** $\frac{7}{25}$	**h)** $\frac{119}{169}$
i) $\frac{3}{4}$	**j)** $\frac{56}{33}$	**k)** $\frac{16}{63}$	**l)** $\frac{24}{7}$

Q2.

a) $\frac{77}{\sqrt{5330}}$	**b)** $\frac{17}{\sqrt{5330}}$	**c)** $\frac{1}{\sqrt{5330}}$	**d)** $\frac{71}{\sqrt{5330}}$
e) $\frac{40}{41}$	**f)** $\frac{63}{130}$	**g)** $\frac{-9}{41}$	**h)** $\frac{16}{65}$
i) $\frac{5}{4}$	**j)** 73	**k)** $\frac{17}{71}$	**l)** $\frac{-40}{9}$

Q3.

a) $\frac{25}{\sqrt{689}}$	**b)** $\frac{-17}{\sqrt{689}}$	**c)** $\frac{-8}{\sqrt{689}}$	**d)** $\frac{20}{\sqrt{689}}$
e) $\frac{12}{13}$	**f)** $\frac{28}{53}$	**g)** $\frac{5}{13}$	**h)** $\frac{-45}{53}$
i) $\frac{2}{3}$	**j)** $\frac{-25}{8}$	**k)** $\frac{-17}{20}$	**l)** $\frac{12}{5}$

Q4.

a) $\frac{3\sqrt{3}+4\sqrt{10}}{\sqrt{2071}}$	**b)** $\frac{3\sqrt{3}-4\sqrt{10}}{\sqrt{2071}}$	**c)** $\frac{12-\sqrt{30}}{\sqrt{2071}}$	**d)** $\frac{12+\sqrt{30}}{\sqrt{2071}}$
e) $\frac{8\sqrt{3}}{19}$	**f)** $\frac{6\sqrt{10}}{109}$	**g)** $\frac{13}{19}$	**h)** $\frac{-1}{109}$
i) $\frac{\sqrt{3}}{4}$	**j)** $\frac{3\sqrt{3}+4\sqrt{10}}{12-\sqrt{30}} = \frac{3\sqrt{10}+4\sqrt{3}}{6}$		

k) $\frac{3\sqrt{3}-4\sqrt{10}}{12+\sqrt{30}} = \frac{4\sqrt{3}-3\sqrt{10}}{6}$ **l)** $\frac{8\sqrt{3}}{13}$

Q5. **a)** $\frac{\sqrt{35}+2\sqrt{6}}{2\sqrt{30}}$ **b)** $\frac{2\sqrt{6}-\sqrt{35}}{2\sqrt{30}}$ **c)** $\frac{2\sqrt{10}-\sqrt{21}}{2\sqrt{30}}$ **d)** $\frac{\sqrt{21}+2\sqrt{10}}{2\sqrt{30}}$

e) $\frac{\sqrt{15}}{4}$ **f)** $\frac{4\sqrt{14}}{15}$ **g)** $\frac{1}{4}$ **h)** $\frac{1}{15}$

i) $\frac{\sqrt{3}}{\sqrt{5}}$ **j)** $\frac{\sqrt{35}+2\sqrt{6}}{2\sqrt{10}-\sqrt{21}} = \frac{15\sqrt{15}+16\sqrt{14}}{19}$

k) $\frac{2\sqrt{6}-\sqrt{35}+}{2\sqrt{10}+\sqrt{21}} = \frac{15\sqrt{15}-16\sqrt{14}}{19}$ **l)** $\sqrt{15}$

Q6. **a)** $\sqrt{2}\cos(x-\frac{\pi}{4})$ **b)** $2\cos(x-\frac{5\pi}{3})$

c) $2\cos(x-\frac{\pi}{6})$ **d)** $2\sqrt{2}\cos(x-\frac{\pi}{4})$

Q7. **a)** $5\sin(x-3.7850..)$ **b)** $10\sin(x-5.3558..)$

c) $\sqrt{5}\sin(x-0.4636..)$ **d)** $\sqrt{10}\sin(x-2.8198..)$

Q8. **a)** $5\cos(x-53.1)°$ **b)** $5\cos(x+306.9)°$

c) $5\sin(x-323.1)°$ **d)** $5\sin(x+36.9)°$

Q9. **a)** $\sqrt{2}\cos(2x-\frac{\pi}{4})$ **b)** $\sqrt{2}\cos(2x-\frac{3\pi}{4})$

Q10. **a)** $5\cos(2\theta-5.355..)$ **b)** $\sqrt{5}\sin(2\theta-5.819..)$

Q11. **a)** $13\cos(x-22.6)°$ **b)** $25\cos(x-286.3)°$

Q12. **a)** $\sqrt{2}\cos(x-45)°$ $x = 0°,\ 90°,\ 360°$

b) $5\cos(x-53.1)°$ $x = 53.1°$

c) $15\cos(x-143.1)°$ $x = 94.9°,\ 191.3°$

d) $\sqrt{13}\cos(x-56.3)°$ $x = 162.4°,\ 310.2°$

Q13. **a)** $5\sin(x-53.1)°$ $x = 76.7°,\ 209.5°$

b) $5\cos(x-233.1)°$ $x = 323.1°$

c) $10\cos(x-53.1)°$ $x = 143.1°$

d) $2\cos(x-210)°$ $x = 0°,\ 240°,\ 360°$

Q14. $10\cos(x+323.1)°$

Q15. $\sqrt{29}\sin(x-68.2)°$

Q16. **a)** $\sqrt{10}\sin(x-71.6)°$ **b)** $x = 161.6°$

Q17. $x = 68.1985\ldots°,\quad 248.1985\ldots°$

Q18. $x = 79.792\ldots°,\quad 100.207\ldots°$

Q19. **a)** PROOF **b)** $\sqrt{17}\cos(x+49.1)°$ **c)** $x = 18.7°,\quad 243.1°$

Q20. **a)** $\sqrt{13}\cos(x-56.3)°$ **b)** $x = 138.3°,\quad 343.3°$

Q21. **a)** PROOF **b)** PROOF **c)** PROOF

d) PROOF **e)** PROOF **f)** PROOF

Q22. PROOF

Q23.

	i)	ii) Max	Min
a)	PROOF	$\mathbf{8}$	$\mathbf{0}$
b)	PROOF	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$
c)	PROOF	$\frac{1}{2}$	$-\frac{1}{2}$
d)	PROOF	3	-3
e)	PROOF	$\frac{1}{4}$	$-\frac{1}{4}$
f)	PROOF	$\frac{3}{2}$	$-\frac{3}{2}$

Q24.

a) $x = 45°$

b) $x = 60°,\ 180°,\ 300°$

c) $x = 10°,\ 50°,\ 130°,\ 170°$

d) $x = 30°,\ 60°$

e) $x = 30°,\ 60°$

f) $x = 20.7°,\ 159.3°,\ 200.7°,\ 339.3°$

g) $x = 120.95°,\ 149.05°$

h) $x = 76.2°,\ 463.8°$

i) $x = 420°,\ 660°$

j) $x = 30°,\ 150°,\ 210°,\ 330°,\ 390°,\ 510°,\ 570°,\ 690°$

k) $x = 0.056,\ 0.991,\ 2.150,\ 3.086,\ 4.245,\ 5.180$

l) $x = 0.421,\ 2.721,\ 3.562,\ 5.863$

m) $x = 0.524,\ 1.047$

n) $x = -2.526, -0.615,\ 0.615,\ 2.526$

o) $x = 1.655,\ 3.058$

p) $x = 0.478,\ 1.093$

q) $x = 1.262,\ 1.880,\ 3.356,\ 3.974,\ 5.450,\ 6.069$

r) $x = 1.090,\ 2.052,\ 4.231,\ 5.194,\ 7.373,\ 8.335$

Q25.

a) $x = 38.6°,\ 121.4°$

b) $x = 5°,\ 125°$

c) $x = 56.8°,\ 153.2°$

d) $x = 56.5°,\ 265.5°$

e) $x = -22.9°,\ 50.9°,\ 337.1°,\ 410.9°$

f) $x = 162.6°,\ 323.4°,\ 522.6°,\ 683.4°$

g) $x = 8°,\ 308°$

h) $x = 133°$

i) $x = 3.6°,\ 86.4°$

j) $x = 18.2°,\ 281.8°$

Q26.

a) $x = 30°,\ 90°,\ 150°,\ 270°$

b) $x = 0°,\ 180°,\ 360°$

c) $x = 60°,\ 180°.\ 300°$

d) $x = 90°,\ 210°,\ 330°$

e) $x = 90°,\ 120°,\ 240°,\ 270°$

f) $x = 210°,\ 330°$

g) $x = 0°,\ 104.5°,\ 255.5°,\ 360°$

h) $x = 30°,\ 150°,\ 199.5°,\ 340.5°$

i) $x = 0.927\ldots\ ,\ 5.356\ldots\ ,\ \frac{2\pi}{3}\ ,\frac{4\pi}{3}$

j) $x = 0.841\ldots\ ,1.823\ldots\ ,4.460\ldots\ ,5.442\ldots$

PROOFS

Q2 1a) $\sin 2A = 2\sin A\cos A$

LHS $\sin 2A = \sin(A+A)$

$= \sin A\cos A + \sin A\cos A$

$= 2\sin A\cos A$, LHS = RHS as required.

b) $\cos 2A = \cos^2 A - \sin^2 A$

LHS $\cos 2A = \cos(A+A)$

$= \cos A.\cos A - \sin A\sin A$

$= \cos^2 A - \sin^2 A$

LHS = RHS as required

c) $\cos 2A = 2\cos^2 A - 1$

$\cos 2A = \cos^2 A - \sin^2 A$

We know $\cos^2 A + \sin^2 A = 1 \Rightarrow \sin^2 A = 1 - \cos^2 A$

$\Rightarrow \cos 2A = \cos^2 A - (1 - \cos^2 A)$

$= \cos^2 A - 1 + \cos^2 A$

$= 2\cos^2 A - 1$

LHS = RHS as required.

d) $\cos 2A = 1 - 2\sin^2 A$

$\cos 2A = \cos^2 A - \sin^2 A$

We know $\cos^2 A + \sin^2 A = 1 \qquad \cos^2 A = 1 - \sin^2 A$.

$\Rightarrow \cos 2A = 1 - \sin^2 A - \sin^2 A$

$= 1 - 2\sin^2 A$

LHS = RHS as required.

e) $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$

We know $\cos 2A = 2\cos^2 A - 1$ * from (c).

$\Rightarrow 2\cos^2 A = 1 + \cos 2A$

$\Rightarrow \cos^2 A = \frac{1}{2}(1 + \cos 2A)$

LHS = RHS as required.

f) $\sin^2 A = \frac{1}{2}(1 - \cos 2A)$.

* We know $\cos 2A = 1 - 2\sin^2 A$ from (d)

$\Rightarrow 2\sin^2 A = 1 - \cos 2A$

$\sin^2 A = \frac{1}{2}(1 - \cos 2A)$

LHS = RHS as required.

Q22.

a) $\frac{\tan A . \cos A}{\sin A} = 1$

$$\frac{\tan A . \cos A}{\sin A} = \frac{\frac{\sin A}{\cos A} . \cos A}{\sin A}$$

$$= \frac{\sin A}{\sin A}$$

$$= 1$$

LHS = RHS as required

b) $1 + \frac{1}{\tan^2 A} = \frac{1}{\sin^2 A}$

$$1 + \frac{1}{\tan^2 A} = 1 + \frac{1}{\sin^2 A / \cos^2 A}$$

$$= 1 + \frac{\cos^2 A}{\sin^2 A}$$

$$= \frac{\sin^2 A}{\sin^2 A} + \frac{\cos^2 A}{\sin^2 A}$$

$$= \frac{\sin^2 A + \cos^2 A}{\sin^2 A}$$

$$= \frac{1}{\sin^2 A}$$

LHS = RHS as required.

Q22 c) $1 + \tan^2 A = \frac{1}{\cos^2 A}$

$1 + \tan^2 A = 1 + \frac{\sin^2 A}{\cos^2 A}$

$= \frac{\cos^2 A}{\cos^2 A} + \frac{\sin^2 A}{\cos^2 A}$

$= \frac{\cos^2 A + \sin^2 A}{\cos^2 A}$

$= \frac{1}{\cos^2 A}$

LHS = RHS as required.

d) $\tan A - \tan B = \frac{\sin(A-B)}{\cos A \cos A}$

$\Rightarrow \tan A - \tan B = \frac{\sin A}{\cos A} - \frac{\sin B}{\cos B}$

$= \frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B}$

$= \frac{\sin(A-B)}{\cos A \cos B}$

LHS = RHS as required

e) $\tan 3A + \tan A = \frac{\sin 4A}{\cos 3A . \cos A}$

$\tan 3A + \tan A = \frac{\sin 3A}{\cos 3A} + \frac{\sin A}{\cos A}$

$= \frac{\sin 3A \cos A + \cos 3A . \sin A}{\cos 3A . \cos A}$.

$= \frac{\sin(3A + A)}{\cos 3A . \cos A}$

$= \frac{\sin(4A)}{\cos 3A . \cos A}$.

f). $\dfrac{\tan^2 A - 1}{\tan^2 A + 1} = 1 - 2\cos^2 A$.

$$\frac{\tan^2 A - 1}{\tan^2 A + 1} = \frac{\frac{\sin^2 A}{\cos^2 A} - 1}{\frac{\sin^2 A}{\cos^2 A} + 1}$$

× top + bottom by $\cos^2 A$

$$= \frac{\sin^2 A - \cos^2 A}{\sin^2 A + \cos^2 A}$$

$$= \sin^2 A - \cos^2 A$$

$$= (1 - \cos^2 A) - \cos^2 A$$

$$= 1 - 2\cos^2 A$$

LHS = RHS as required.

g) $\dfrac{\tan^2 A}{\sin^2 A} - 1 = \tan^2 A$

$$\frac{\tan^2 A}{\sin^2 A} - 1 = \frac{\tan^2 A - \sin^2 A}{\sin^2 A}$$

$$= \frac{\frac{\sin^2 A}{\cos^2 A} - \sin^2 A}{\sin^2 A}$$

$$= \frac{1}{\cos^2 A} - 1$$

$$= \frac{1 - \cos^2 A}{\cos^2 A}$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

LHS = RHS as required.

h) $\sin A - \sin A \cos^2 A = \sin^3 A$

$$\sin A - \sin A \cos^2 A = \sin A\,(1 - \cos^2 A)$$
$$= \sin A\,(\sin^2 A)$$
$$= \sin^3 A$$

LHS = RHS as required.

i) $\dfrac{\cos A}{1 - \sin A} - \dfrac{\cos A}{1 + \sin A} = 2 \tan A$

$$\frac{\cos A}{1 - \sin A} - \frac{\cos A}{1 + \sin A} = \frac{(1+\sin A)\cos A - (1 - \sin A)\cos A}{1 - \sin^2 A}$$
$$= \frac{\cos A\,(1 + \sin A - 1 + \sin A)}{\cos^2 A}$$
$$= \frac{\cos A\,(2 \sin A)}{\cos^2 A}$$
$$= \frac{2 \sin A}{\cos A}$$
$$= 2 \tan A$$

LHS = RHS as required.

j) $(\sin x - \cos x)^2 + (\sin x + \cos x)^2 = 2$

$$(\sin x - \cos x)^2 + (\sin x + \cos x)^2 = \sin^2 x - 2\sin x \cos x + \cos^2 x + \sin^2 x + 2\sin x \cos x + \cos^2 x$$
$$= 2\sin^2 x + 2\cos^2 x$$
$$= 2(\sin^2 x + \cos^2 x)$$
$$= 2$$

LHS = RHS as required

k) $\frac{1-\sin x}{\cos x} = \frac{\cos x}{1+\sin x}$

$$\frac{1-\sin x}{\cos x} = \frac{\cos x(1-\sin x)}{\cos^2 x}$$

$$= \frac{\cos x(1-\sin x)}{1-\sin^2 x}$$

$$= \frac{\cos x(1-\sin x)}{(1-\sin x)(1+\sin x)}$$

$$= \frac{\cos x}{1+\sin x}$$

LHS = RHS as required.

l) $\sin^4 x - \cos^4 x = 1 - 2\cos^2 x$

$$\sin^4 x - \cos^4 x = (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)$$

$$= \sin^2 x - \cos^2 x$$

$$= 1 - \cos^2 x - \cos^2 x$$

$$= 1 - 2\cos^2 x$$

LHS = RHS as required.

Q23 a i) $(\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2\sin x\cos x$

$= 1 + 2\sin x\cos x$

$= 1 + \sin 2x$

b i) $\sin^3 x.\cos x + \sin x.\cos^3 x = \sin x\cos x(\sin^2 x + \cos^2 x)$

$= \sin x.\cos x$

$= \frac{1}{2}(2\sin x\cos x)$

$= \frac{1}{2}\sin 2x$

c i) $\sin 3x = \sin(2x + x)$

$= \sin 2x\cos x + \cos 2x\sin x$

$= 2\sin x.\cos^2 x + \sin x(1 - 2\sin^2 x)$

$= \sin x(2\cos^2 x + 1 - 2\sin^2 x)$

$= \sin x(2 - 2\sin^2 x + 1 - 2\sin^2 x)$

$= 3\sin x - 4\sin^3 x$

d i) $\cos 3x = \cos(2x + x)$

$= \cos 2x.\cos x - \sin 2x.\sin x$

$= (2\cos^2 A - 1)\cos x - 2\sin^2 x\cos x$

$= \cos x(2\cos^2 A - 1 - 2\sin^2 x)$

$= \cos x(2\cos^2 x - 1 - (2 - 2\cos^2 x))$

$= 4\cos^3 x - 3\cos x$

Q23 continued

e) i)

$$\frac{2\tan x}{1+\tan^2 x} = \frac{2 \cdot \frac{\sin x}{\cos x}}{1 + \frac{\sin^2 x}{\cos^2 x}}$$

$$= \frac{2 \cdot \frac{\sin x}{\cos x}}{\frac{\cos^2 x + \sin^2 x}{\cos^2 x}}$$

$$= \frac{2 \sin x}{1/\cos x}$$

$$= 2\sin x . \cos x$$

$$= \sin 2x$$

f) i)

$$\frac{1-\tan^2 x}{1+\tan^2 x} = \frac{1 - \frac{\sin^2 x}{\cos^2 x}}{1 + \frac{\sin^2 x}{\cos^2 x}}$$

$$= \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x}$$

$$= \cos^2 x - \sin^2 x$$

$$= \cos 2x.$$