Higher - Expressions & Formulae Solutions

24 November 2017 11:06

Outcome 1.1

1) a) $\log_4 2 + \log_4 8$

$= \log_4 16$

$= 2$

b) $\log_3 108 - \log_3 4$

$= \log_3 27$

$= 3$

c) $\log_3 18 - \log_3 2$

$= \log_3 9$

$= 2$

d) $\log_5 100 - \log_5 4$

$= \log_5 25$

$= 2$

e) $\log_4 8 + \log_4 8$

$= \log_4 64$

$= 3$

f) $2\log_{10} 2 + 2\log_{10} 5$

$= \log_{10} 2^2 + \log_{10} 5^2$

$= \log_{10} 4 + \log_{10} 25$

$= \log_{10} 100$

$= 2$

g) $\log_9 3 - \log_9 6 + \log_9 18$

$= \log_9 \frac{1}{2} + \log_9 18$

$= \log_9 9$

$= 1$

h) $\log_3 9 - \log_3 \frac{1}{3}$

$= \log_3 27$

$= 3$

i) $\frac{1}{2}\log_2 16 - \frac{1}{3}\log_2 8$

$= \log_2 16^{\frac{1}{2}} - \log_2 8^{\frac{1}{3}}$

$= \log_2 4 - \log_2 2$

$= \log_2 2$

$= 1$

2) a) $\log_4(x+3) = 2$

$x+3 = 4^2$

$x+3 = 16$

$x = 13$

b) $\log_3(x-2) = 4$

$x-2 = 3^4$

$x-2 = 81$

$x = 83$

c) $\log_6(x-8) = 2$

$x-8 = 6^2$

$x-8 = 36$

$x = 44$

d) $\log_a 4 + \log_a x = \log_a 12$

$\log_a 4x = \log_a 12$

$4x = 12$

$x = 3$

e) $2\log_a 3 + \log_a x = \log_a 36$

$\log_a 3^2 + \log_a x = \log_a 36$

$\log_a 9x = \log_a 36$

$9x = 36$

$x = 4$

f) $\log_a(2x+1) + \log_a 3x = \log_a 63$

$\log_a\left(3x(2x+1)\right) = \log_a 63$

$3x(2x+1) = 63$

$6x^2 + 3x = 63$

$6x^2 + 3x - 63 = 0$

$2x^2 + x - 21 = 0$

$(2x+7)(x-3) = 0$

$x = -\frac{7}{2}, \; x = 3$

g) $\log_2 x + \log_2(x-3) = 2$

$\log_2\left(x(x-3)\right) = 2$

$x(x-3) = 2^2$

$x(x-3) = 4$

h) $\log_2(x-1) + \log_2(x+1) = 3$

$\log_2\left((x-1)(x+1)\right) = 3$

$(x-1)(x+1) = 2^3$

$x^2 - 1 = 8$

i) $\log_3 6x - \log_3(x-2) = 2$

$\log_3\left(\frac{6x}{x-2}\right) = 2$

$\frac{6x}{x-2} = 3^2$

$\log_2 (x(x$ ...

$$x(x-3) = 2^2$$
$$x(x-3) = 4$$
$$x^2 - 3x - 4 = 0$$
$$(x-4)(x+1) = 0$$
$$x = 4 \ , \ x = -1$$

$$(x-1)(x+1) = 2^3$$
$$x^2 - 1 = 8$$
$$x^2 - 9 = 0$$
$$(x+3)(x-3) = 0$$
$$x = -3 \ , \ x = 3$$

$$\frac{6x}{x-2} = 3^2$$
$$\frac{6x}{x-2} = 9$$
$$6x = 9(x-2)$$
$$6x = 9x - 18$$
$$-3x = -18$$
$$x = 6$$

3) $2\log_m n = \log_m 16 + 1$ $\rightarrow$ target : $n = 4\sqrt{m}$

$$\log_m n^2 = \log_m 16 + 1$$
$$\log_m n^2 - \log_m 16 = 1$$
$$\log_m \left(\frac{n^2}{16}\right) = 1$$
$$\frac{n^2}{16} = m^1$$
$$n^2 = 16m$$
$$n = \sqrt{16m}$$
$$n = 4\sqrt{m}$$ as required.

4) a) $M = M_0 e^{-kt}$

$$8 = 10e^{-k(5)}$$
$$0.8 = e^{-5k}$$
$$\log_e 0.8 = \log_e e^{-5k}$$
$$\log_e 0.8 = -5k$$
$$\frac{\log_e 0.8}{-5} = k$$
$$k = 0.0446$$

b) $5 = 10e^{-0.0446t}$

$$0.5 = e^{-0.0446t}$$
$$\log_e 0.5 = \log_e e^{-0.0446t}$$
$$\log_e 0.5 = -0.0446t$$
$$\frac{\log_e 0.5}{-0.0446} = t$$
$t = 15.54$ years.

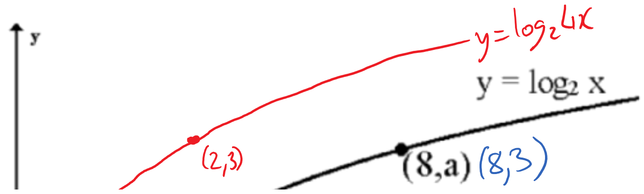
5) a) $y(t) = Ae^{kt}$

$$800 = 500e^{k(24)}$$
$$1.6 = e^{24k}$$
$$\log_e 1.6 = \log_e e^{24k}$$
$$\log_e 1.6 = 24k$$
$$\frac{\log_e 1.6}{24} = k$$
$$k = 0.0196$$

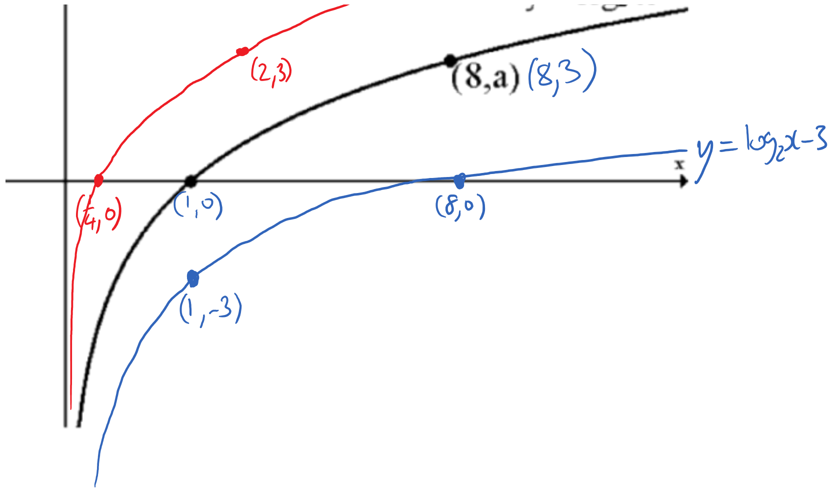
b) $1000 = 500e^{0.0196t}$

$$2 = e^{0.0196t}$$
$$\log_e 2 = \log_e e^{0.0196t}$$
$$\log_e 2 = 0.0196t$$
$$\frac{\log_e 2}{0.0196} = t$$
$t = 35.4$ hours

6) a) $y = \log_2 x$ b) c)



$a = \log_2 8$
$a = 3$



7) $m = \dfrac{7-4}{6-0}$

$m = \dfrac{3}{6}$

$m = \frac{1}{2}$

$c = 4$

$y = \frac{1}{2}x + 4$

so $\log_2 y = \frac{1}{2}\log_2 x + 4$

$\log_2 y = \log_2 x^{1/2} + 4$

$\log_2 y - \log_2 x^{1/2} = 4$

$\log_2 \dfrac{y}{x^{1/2}} = 4$

$\dfrac{y}{x^{1/2}} = 2^4$

$\dfrac{y}{x^{1/2}} = 16$

$y = 16x^{1/2}$ $\quad\underline{\underline{k = 16,\ n = \frac{1}{2}}}$

8) $m = \dfrac{6-0}{0-2}$

$m = \dfrac{6}{-2}$

$m = -3$

$c = 6$

$y = -3x + 6$

so $\log_2 y = -3x + 6$

$\log_2 y = -3x\log_2 2 + 6$

$\log_2 y = \log_2 2^{-3x} + 6$

$\log_2 y - \log_2 2^{-3x} = 6$

$\log_2 \dfrac{y}{2^{-3x}} = 6$

$\dfrac{y}{2^{-3x}} = 2^6$

$\dfrac{y}{2^{-3x}} = 64$

$y = 64 \times 2^{-3x}$

$y = 64 \times \frac{1}{8}^x$ $\quad\underline{\underline{a = 64,\ b = \frac{1}{8}}}$

9) $P_t = P_0 e^{-kt}$

a) $1 = 2e^{-k(25)}$

$0.5 = e^{-25k}$

$\log_e 0.5 = \log_e e^{-25k}$

$\log_e 0.5 = -25k$

$\dfrac{\log_e 0.5}{-25} = k$

$k = 0.0277$

b) Let initial value $= 100$

$P_t = 100\, e^{-0.0277(80)}$

$= 10.904$

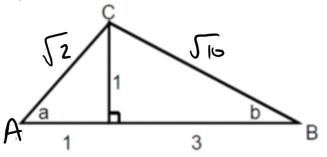
decrease $= 100 - 10.904 = 89.096$

% decrease $= \dfrac{89.096}{100} \times 100 = \underline{\underline{89.096\%}}$

Outcome $\underline{1.2}$

Outcome 1.2

1)

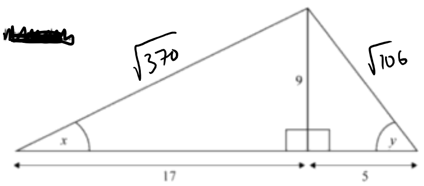


$\sin a = \frac{1}{\sqrt{2}}$ $\quad \sin b = \frac{1}{\sqrt{10}}$

$\cos a = \frac{1}{\sqrt{2}}$ $\quad \cos b = \frac{3}{\sqrt{10}}$

$$\sin(a+b) = \sin a\cos b + \cos A \sin B$$
$$= \frac{1}{\sqrt{2}} \times \frac{3}{\sqrt{10}} + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{10}}$$
$$= \frac{3}{\sqrt{20}} + \frac{1}{\sqrt{20}}$$
$$= \frac{4}{\sqrt{20}}$$
$$= \frac{4}{2\sqrt{5}}$$
$$= \frac{2}{\sqrt{5}} \text{ as required}$$

2)



$\sin x = \frac{9}{\sqrt{370}}$ $\quad \sin y = \frac{9}{\sqrt{106}}$

$\cos x = \frac{17}{\sqrt{370}}$ $\quad \cos y = \frac{5}{\sqrt{106}}$

$$\sin(x-y) = \sin x\cos y - \cos x \sin y$$
$$= \frac{9}{\sqrt{370}} \times \frac{5}{\sqrt{106}} - \frac{17}{\sqrt{370}} \times \frac{9}{\sqrt{106}}$$
$$= \frac{45}{\sqrt{39220}} - \frac{153}{\sqrt{39220}}$$
$$= \frac{-108}{\sqrt{39220}}$$

3)
$$LHS = (3+2\cos x)(3-2\cos x)$$
$$= 9 - 6\cos x + 6\cos x - 4\cos^2 x$$
$$= -4\cos^2 x + 9$$
$$= -4(1-\sin^2 x) + 9$$
$$= -4 + 4\sin^2 x + 9$$
$$= 4\sin^2 x + 5 = RHS$$

4)
$$LHS = 3\cos 2x - 4\cos^2 x$$
$$= 3(1-2\sin^2 x) - 4(1-\sin^2 x)$$
$$= 3 - 6\sin^2 x - 4 + 4\sin^2 x$$
$$= -1 - 2\sin^2 x$$
$$= RHS$$

5)
$$2\sin x + 3\cos x = k\sin(x+a)^\circ$$
$$= k\sin x\cos a + k\cos x \sin a$$

$k\cos a = 2$

$k\sin a = 3$

S | A

T | C

$\tan^{-1}\left(\frac{3}{2}\right) =$

$\tan a^\circ = \frac{k\sin a}{k\cos a}$

$= \frac{3}{2}$

$a^\circ = 56.3^\circ$

$k = \sqrt{3^2+2^2}$

$k = \sqrt{13}$

$$\therefore 2\sin x + 3\cos x = \sqrt{13}\sin(x+56.3)^\circ$$

6)
$$\cos x - \sin x = k\cos(x-\alpha)$$
$$= k\cos x\cos\alpha + k\sin x\sin\alpha$$

$k\cos\alpha = 1$

$k\sin\alpha = -1$

S | A

T | C

$\tan^{-1}(1) = 45^\circ$

$\tan\alpha^\circ = \frac{k\sin a}{k\cos a}$

$= \frac{-1}{1}$

$= -1$

$\alpha = 315^\circ$

$k = \sqrt{1^2+(-1)^2}$

$k = \sqrt{2}$

$\therefore \cos x - \sin x = \sqrt{2}\cos(x-315)^\circ$

7) a) $5\cos x - 12\sin x = k\sin(x-a)$

$= k\sin x\cos a - k\cos x\sin a$

$k\cos a = -12$

$-k\sin a = 5$

$k\sin a = -5$

$k = \sqrt{(-12)^2+(-5)^2}$

$k = 13$

S	A
T ✓	C ✓

$\tan^{-1}\left(\frac{5}{12}\right) = 22.6^\circ$

$\tan a = \frac{k\sin a}{k\cos a}$

$= \frac{-5}{-12}$

$a = 180 + 22.6$

$= 202.6^\circ$

$\therefore 5\cos x - 12\sin x = 13\sin(x-202.6)^\circ$

b) max value of $13\sin(x-202.6)^\circ = 13$

8) $4\sin x + 3\cos x = 2.5 \qquad 0 \le x \le 180$

write in wave equation form.

$4\sin x + 3\cos x = k\cos(x-a)$

$= k\cos x\cos a + k\sin x\sin a$

$k\cos a = 3$

$k\sin a = 4$

$k = \sqrt{4^2+3^2}$

$k = \sqrt{25}$

$k = 5$

S	A ✓
T	C ✓

$\tan^{-1}\left(\frac{4}{3}\right) = 53.1$

$\tan a^\circ = \frac{k\sin a}{k\cos a}$

$= \frac{4}{3}$

$a^\circ = 53.1^\circ$

$\therefore 4\sin x + 3\cos x = 5\cos(x-53.1)^\circ$

so $5\cos(x-53.1)^\circ = 2.5$

$\cos(x-53.1)^\circ = 0.5$

$x - 53.1 = 60^\circ, 300^\circ$

$x = 113.1^\circ, 353.1^\circ$

S	A ✓
T	C ✓

$\cos^{-1}(0.5) = 60^\circ$

9) a) i) $3x - 2y = 0$

$2y = 3x$

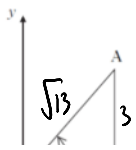
$y = \frac{3}{2}x$

$\therefore m = \frac{3}{2}$

$m = \tan\theta$

$\tan a = \frac{3}{2}$ $\left(\frac{\text{vertical}}{\text{horizontal}}\right)$

ii) $\sin a = \frac{3}{\sqrt{13}} \qquad \cos a = \frac{2}{\sqrt{13}}$



ii) $\sin a = \frac{3}{\sqrt{13}}$ $\quad \cos a = \frac{2}{\sqrt{13}}$

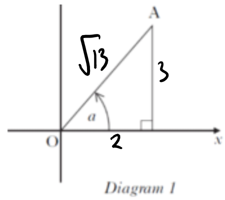


Diagram 1

b) $3x - 4y = 0$

$4y = 3x$

$y = \frac{3}{4}x$

$m = \frac{3}{4}$ $\left(\frac{v}{h}\right)$

$\sin b = \frac{3}{5}$ $\quad \cos b = \frac{4}{5}$

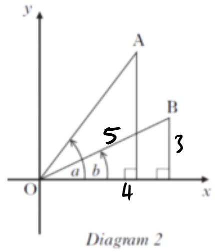


Diagram 2

c) i) $\sin(a-b) = \sin a \cos b - \cos a \sin b$

$= \frac{3}{\sqrt{13}} \times \frac{4}{5} - \frac{2}{\sqrt{13}} \times \frac{3}{5}$

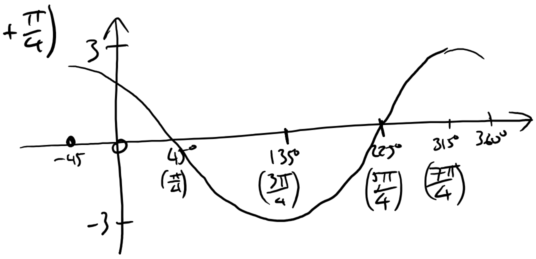
$= \frac{12}{5\sqrt{13}} - \frac{6}{5\sqrt{13}}$

$\underline{\underline{\frac{6}{5\sqrt{13}}}}$

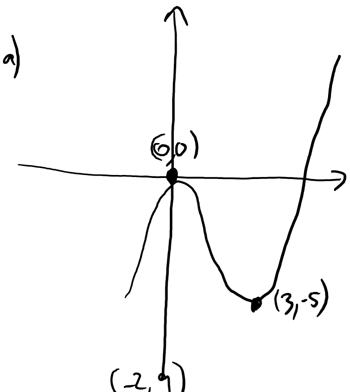
ii) $\sin(b-a) = \underline{\underline{\frac{-6}{5\sqrt{13}}}}$

Outcome 1.3

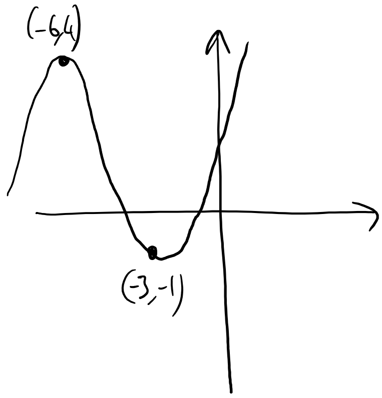
1) $y = 3\cos\left(x + \frac{\pi}{4}\right)$



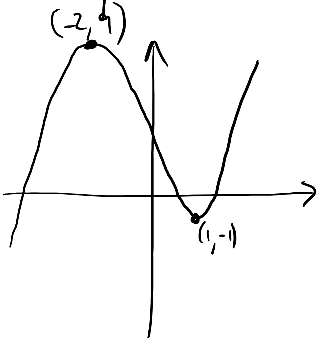
2) a)



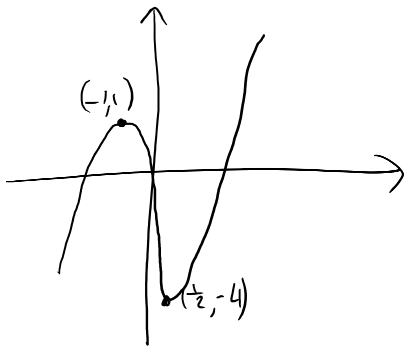
b)



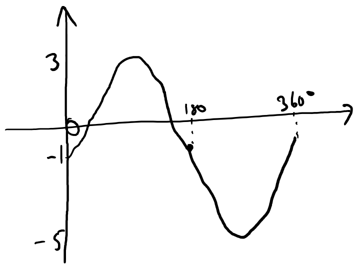
c)



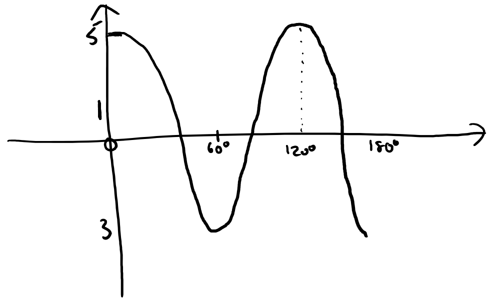
d)



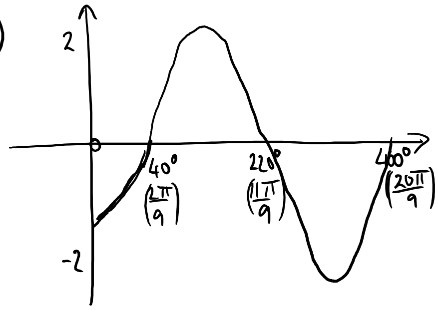
3) a)



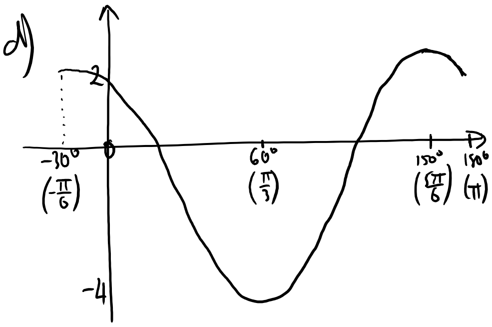
b)



c)



d)



4) a) $y = 4\cos x + 2$ b) $y = 4\sin 2x + 3$

5) a) $y = \log_2 x$

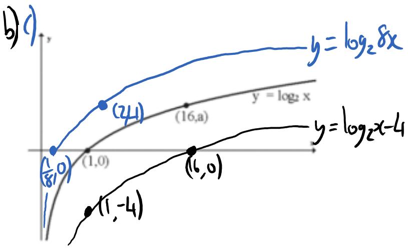
$a = \log_2 16$

$2^a = 16$

$2^a = 2^4$

$\underline{\underline{a = 4}}$

b) i)



6) $y = \log_b(x+a)$

$(4,0)$: $0 = \log_b(4+a)$
x y

$b^0 = (4+a)$

$1 = 4 + a$

$\underline{\underline{a = -3}}$

$(19,4)$ $y = \log_b(x-3)$
x, y

$4 = \log_b(19-3)$

$4 = \log_b 16$

$b^4 = 16$

$\underline{\underline{b = 2}}$ since $b > 0$

7) $f(x) = 2x^2$ $g(x) = 5x - 4$

a) $g(2) = 5(2) - 4 = 6$ $f(g(2)) = f(6) = 2(6)^2$

$= 2 \times 36$

$\underline{\underline{= 72}}$

b) $f(g(x)) = f(5x-4)$

$= 2(5x-4)^2$

$= 2(5x-4)(5x-4)$

$= 2(25x^2 - 40x + 16)$

$\underline{\underline{= 50x^2 - 80x + 32}}$

8) $f(g(x)) = f(x^2+3)$

$= (x^2+3-1)(x^2+3+3)$

$= (x^2+2)(x^2+6)$

$\underline{\underline{= x^4 + 8x^2 + 12}}$

$g(g(x)) = g(x^2+3)$

$= (x^2+3)^2 + 3$

$= x^4 + 6x^2 + 9 + 3$

$\underline{\underline{= x^4 + 6x^2 + 12}}$

$f(g(x)) - g(g(x))$

$= (x^4 + 8x^2 + 12) - (x^4 + 6x^2 + 12)$

$\underline{\underline{= 2x^2}}$

9) $f(x) = \frac{1}{x^2 - 4}$ $g(x) = x + 1$

a) $h(x) = f(g(x)) = f(x+1)$

$= \frac{1}{(x+1)^2 - 4}$

$= \frac{1}{x^2 + 2x + 1 - 4}$

$= \frac{1}{x^2 + 2x - 3}$

b) $x^2 + 2x - 3 \neq 0$

$(x+3)(x-1) \neq 0$

$\underline{\underline{x \neq -3,\ x \neq 1}}$

10) $f(x) = 3x-2$ $\quad g(x) = 3x+2$

a) $f(g(x)) = f(3x+2)$
$= 3(3x+2)-2$
$= 9x+6-2$
$= \underline{9x+4}$

$g(f(x)) = g(3x-2)$
$= 3(3x-2)+2$
$= 9x-6+2$
$= \underline{9x-4}$

b) $f(g(x)) \times g(f(x))$
$= (9x+4)(9x-4)$
$= 81x^2-16$

Least value when $x^2=0$
$\therefore$ min value is $\underline{\underline{-16}}$

11) a) $f(x) = 4x-5$
$y = 4x-5$
let $x = 4y-5$
$x+5 = 4y$
$\frac{x+5}{4} = y$
so $f^{-1}(x) = \underline{\underline{\frac{x+5}{4}}}$

b) $f(x) = \frac{x}{6}$
$y = \frac{x}{6}$
let $x = \frac{y}{6}$
$6x = y$
so $f^{-1}(x) = \underline{\underline{6x}}$

c) $f(x) = \frac{2x}{5}+4$
$y = \frac{2x}{5}+4$
let $x = \frac{2y}{5}+4$
$x-4 = \frac{2y}{5}$
$5(x-4) = 2y$
$\frac{5(x-4)}{2} = y$
so $f^{-1}(x) = \underline{\underline{\frac{5(x-4)}{2}}}$

d) $f(x) = \frac{2x-5}{4}$
$y = \frac{2x-5}{4}$
let $x = \frac{2y-5}{4}$
$4x = 2y-5$
$4x+5 = 2y$
$\frac{4x+5}{2} = y$
so $f^{-1}(x) = \underline{\underline{\frac{4x+5}{2}}}$

e) $f(x) = \frac{4x+7}{2}$
$y = \frac{4x+7}{2}$
let $x = \frac{4y+7}{2}$
$2x = 4y+7$
$2x-7 = 4y$
$\frac{2x-7}{4} = y$
so $f^{-1}(x) = \underline{\underline{\frac{2x-7}{4}}}$

f) $f(x) = 12-\frac{3}{4}x$
$y = 12-\frac{3}{4}x$
let $x = 12-\frac{3}{4}y$
$x-12 = -\frac{3}{4}y$
$4(x-12) = -3y$
$\frac{4(x-12)}{-3} = y$
so $f^{-1}(x) = \underline{\underline{\frac{4(x-12)}{-3}}}$

g) $f(x) = \frac{8-3x}{13}$
$y = \frac{8-3x}{13}$
let $x = \frac{8-3y}{13}$
$13x = 8-3y$
$13x-8 = -3y$
$\frac{13x-8}{-3} = y$
so $f^{-1}(x) = \underline{\underline{\frac{13x-8}{-3}}}$

h) $f(x) = \frac{-3x+4}{-9}$
$y = \frac{-3x+4}{-9}$
let $x = \frac{-3y+4}{-9}$
$-9x = -3y+4$
$-9x-4 = -3y$
$\frac{-9x-4}{-3} = y$
$\frac{9x+4}{3} = y$
so $f^{-1}(x) = \underline{\underline{\frac{9x+4}{3}}}$

Outcome 1.4

1) $\vec{AB} = \underline{b} - \underline{a}$
$= \begin{pmatrix}4\\0\\2\end{pmatrix} - \begin{pmatrix}1\\2\\0\end{pmatrix}$
$= \begin{pmatrix}3\\-2\\2\end{pmatrix}$

$\vec{BC} = \underline{c} - \underline{b}$
$= \begin{pmatrix}13\\-6\\8\end{pmatrix} - \begin{pmatrix}4\\0\\2\end{pmatrix}$
$= \begin{pmatrix}9\\-6\\6\end{pmatrix}$
$= 3\begin{pmatrix}3\\-2\\2\end{pmatrix} = 3\vec{AB}$

since $\vec{BC} = 3\vec{AB}$, lines are parallel.
B is a common point, so points are collinear.
- condition 1 met.
since $\vec{BC} = 3\vec{AB}$, the distance between BC is 3 times AB. $\therefore$ condition 2 is met.
$\therefore$ The flags meet both conditions.

2) $Q = \frac{3}{8}R + \frac{5}{8}P$

$8Q = 3R + 5P$
$8Q = 3\begin{pmatrix}10\\41\\-6\end{pmatrix} + 5\begin{pmatrix}2\\1\\18\end{pmatrix}$
$= \begin{pmatrix}30\\123\\-18\end{pmatrix} + \begin{pmatrix}10\\5\\90\end{pmatrix}$
$= \begin{pmatrix}40\\128\\72\end{pmatrix}$
$Q = \begin{pmatrix}5\\16\\9\end{pmatrix}$ so $\underline{\underline{Q(5,16,9)}}$

3) $\vec{AB} = \underline{b} - \underline{a}$
$= \begin{pmatrix}7\\-6\\9\end{pmatrix} - \begin{pmatrix}0\\-3\\5\end{pmatrix}$
$= \begin{pmatrix}7\\-3\\4\end{pmatrix}$

$\vec{BC} = \underline{c} - \underline{b}$
$= \begin{pmatrix}21\\-12\\17\end{pmatrix} - \begin{pmatrix}7\\-6\\9\end{pmatrix}$
$= \begin{pmatrix}14\\-6\\8\end{pmatrix} = 2\begin{pmatrix}7\\-3\\4\end{pmatrix} = 2\vec{AB}$

since $\vec{BC} = 2\vec{AB}$, lines are parallel.
B is a common point, so points are collinear.
$\vec{AB} : \vec{BC}$
$\underline{\underline{1 : 2}}$

$$= \begin{pmatrix} 7 \\ -3 \\ 4 \end{pmatrix} \qquad = \begin{pmatrix} 14 \\ -6 \\ 8 \end{pmatrix} = 2\begin{pmatrix} 7 \\ -3 \\ 4 \end{pmatrix} = 2\overrightarrow{AB} \qquad \underline{\underline{1:2}}$$

4) $\underline{u} = \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix}$ $\underline{v} = \begin{pmatrix} 1 \\ a \\ \sqrt{7} \end{pmatrix}$

$|\underline{u}| = \sqrt{2^2 + (-2)^2 + 4^2}$
$= \sqrt{24}$

$|\underline{v}| = \sqrt{1^2 + a^2 + \sqrt{7}^2}$
$= \sqrt{a^2 + 8}$

so $\sqrt{a^2+8} = \sqrt{24}$
$a^2 + 8 = 24$
$a^2 = 16$
$\underline{\underline{a = \pm 4}}$

5) $\overrightarrow{BA} = \underline{a} - \underline{b}$
$= \begin{pmatrix} 6 \\ -1 \\ 9 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \\ 11 \end{pmatrix}$
$= \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$

$\overrightarrow{BC} = \underline{c} - \underline{b}$
$= \begin{pmatrix} 7 \\ -8 \\ 14 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \\ 11 \end{pmatrix}$
$= \begin{pmatrix} 4 \\ -6 \\ 3 \end{pmatrix}$

$\overrightarrow{BA}.\overrightarrow{BC} = 3\times4 + 1\times(-6) + (-2)\times3$
$= 12 + (-6) + (-6)$
$= 0$

since $\overrightarrow{BA}.\overrightarrow{BC} = 0$, triangle is right angled at B.

6) a) $\overrightarrow{RS} = \underline{s} - \underline{r}$
$= \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ -1 \end{pmatrix}$
$= \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$

$\overrightarrow{RT} = \underline{t} - \underline{r}$
$= \begin{pmatrix} 6 \\ 1 \\ -2 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ -1 \end{pmatrix}$
$= \begin{pmatrix} 6 \\ -3 \\ -1 \end{pmatrix}$

b) $\overrightarrow{RS}.\overrightarrow{RT} = 1\times6 + 1\times(-3) + 3\times(-1)$
$= 6 + (-3) + (-3)$
$= 0$

c) since $\overrightarrow{RS}.\overrightarrow{RT} = 0$, vectors are perpendicular.

7) a) $\overrightarrow{AS} = \underline{s} - \underline{a}$
$= \begin{pmatrix} 5 \\ 5 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$
$= \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix}$

$\overrightarrow{AT} = \underline{t} - \underline{a}$
$= \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$
$= \begin{pmatrix} 3 \\ -3 \\ 4 \end{pmatrix}$

b) $\overrightarrow{AS}.\overrightarrow{AT} = 4\times3 + 2\times(-3) + 3\times4$
$= 12 + (-6) + 12$
$= 18$

$|\overrightarrow{AS}| = \sqrt{4^2 + 2^2 + 3^2} = \sqrt{29}$

$|\overrightarrow{AT}| = \sqrt{3^2 + (-3)^2 + 4^2} = \sqrt{34}$

$\cos\theta = \dfrac{\overrightarrow{AS}.\overrightarrow{AT}}{|\overrightarrow{AS}||\overrightarrow{AT}|}$
$= \dfrac{18}{\sqrt{29}\sqrt{34}}$
$= 0.573$
$\theta = \underline{\underline{55.0°}}$

8) a) C(11, 12, 6) D(8, 8, 4)

b) $\overrightarrow{CB} = \underline{b} - \underline{c}$
$= \begin{pmatrix} 11 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 11 \\ 12 \\ 6 \end{pmatrix}$
$= \begin{pmatrix} 0 \\ -8 \\ -4 \end{pmatrix}$

$\overrightarrow{CD} = \underline{d} - \underline{c}$
$= \begin{pmatrix} 8 \\ 8 \\ 4 \end{pmatrix} - \begin{pmatrix} 11 \\ 12 \\ 6 \end{pmatrix}$
$= \begin{pmatrix} -3 \\ -4 \\ -2 \end{pmatrix}$

c) $\overrightarrow{CB}.\overrightarrow{CD} = 0\times(-3) + (-8)\times(-4) + (-4)\times(-2)$
$= 0 + 32 + 8$
$= 40$

$|\overrightarrow{CB}| = \sqrt{0^2 + (-8)^2 + (-4)^2} = \sqrt{80}$

$|\overrightarrow{CD}| = \sqrt{(-3)^2 + (-4)^2 + (-2)^2} = \sqrt{29}$

$\cos\theta = \dfrac{\overrightarrow{CB}.\overrightarrow{CD}}{|\overrightarrow{CB}||\overrightarrow{CD}|}$
$= \dfrac{40}{\sqrt{80}\sqrt{29}}$
$= 0.830$
$\theta = \underline{\underline{33.9°}}$

9) a) i) $\overrightarrow{AT} = \underline{t} - \underline{a}$
$= \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix} - \begin{pmatrix} -7 \\ -8 \\ 1 \end{pmatrix}$
$= \begin{pmatrix} 10 \\ 10 \\ 4 \end{pmatrix}$
$= 2\begin{pmatrix} 5 \\ 5 \\ 2 \end{pmatrix}$

$\overrightarrow{TB} = \underline{b} - \underline{t}$
$= \begin{pmatrix} 18 \\ 17 \\ 11 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$
$= \begin{pmatrix} 15 \\ 15 \\ 6 \end{pmatrix}$
$= 3\begin{pmatrix} 5 \\ 5 \\ 2 \end{pmatrix}$

ii) $\overrightarrow{AT} : \overrightarrow{TB}$
$2 : 3$

$$= 2\begin{pmatrix}5\\5\\2\end{pmatrix} \qquad = 3\begin{pmatrix}5\\5\\2\end{pmatrix}$$

b) $\vec{TB} = \begin{pmatrix}15\\15\\6\end{pmatrix}$

$C(x,0,0)$ since on x-axis.

$\vec{TC} = \underline{c} - \underline{t}$

$$= \begin{pmatrix}x\\0\\0\end{pmatrix} - \begin{pmatrix}3\\2\\5\end{pmatrix}$$

$$= \begin{pmatrix}x-3\\-2\\-5\end{pmatrix}$$

$\vec{TB} \cdot \vec{TC} = 0$ for perpendicular vectors

$$\vec{TB} \cdot \vec{TC} = 15(x-3) + 15\times(-2) + 6\times(-5)$$
$$= 5x - 45 + (-30) + (-30)$$
$$= 5x - 105$$

so $5x - 105 = 0$

$5x = 105$

$x = 21$

$\therefore C\ \underline{\underline{(21,0,0)}}$

10) a) i) $\vec{BA} = \underline{a} - \underline{b}$

$$= \begin{pmatrix}3\\-3\\0\end{pmatrix} - \begin{pmatrix}2\\-3\\1\end{pmatrix}$$

$$= \begin{pmatrix}1\\0\\-1\end{pmatrix}$$

$\vec{BC} = \underline{c} - \underline{b}$

$$= \begin{pmatrix}4\\k\\0\end{pmatrix} - \begin{pmatrix}2\\-3\\1\end{pmatrix}$$

$$= \begin{pmatrix}2\\k+3\\-1\end{pmatrix}$$

ii) $\vec{BA} \cdot \vec{BC} = 1\times 2 + 0 + (-1)\times(-1)$

$$= 2 + 1$$
$$= 3$$

$$|\vec{BA}| = \sqrt{1^2 + 0^2 + (-1)^2} = \sqrt{2}$$

$$|\vec{BC}| = \sqrt{2^2 + (k+3)^2 + (-1)^2}$$
$$= \sqrt{4 + k^2 + 6k + 9 + 1}$$
$$= \sqrt{k^2 + 6k + 14}$$

$$\cos\theta = \frac{\vec{BA} \cdot \vec{BC}}{|\vec{BA}||\vec{BC}|}$$
$$= \frac{3}{\sqrt{2}\sqrt{k^2+6k+14}}$$
$$= \underline{\underline{\frac{3}{\sqrt{2(k^2+6k+14)}}}}$$

b) if $\theta = 30°$, then

$$\cos 30° = \frac{3}{\sqrt{2(k^2+6k+14)}}$$

$$\frac{\sqrt{3}}{2} = \frac{3}{\sqrt{2(k^2+6k+14)}}$$

$$\sqrt{3}\sqrt{2(k^2+6k+14)} = 6$$

$$\sqrt{6(k^2+6k+14)} = 6$$

$$6(k^2+6k+14) = 36$$

$$k^2+6k+14 = 6$$

$$k^2+6k+8 = 0$$

$$(k+4)(k+2) = 0$$

$$\underline{\underline{k=-4,\quad k=-2}}$$

11) a) $\vec{DC} = \underline{u}$ b) $\vec{HC} = -\underline{w}$ c) $\vec{AC} = \underline{u} + \underline{v}$ d) $\vec{FD} = \underline{v} - \underline{w}$ e) $\vec{CF} = -\underline{u} - \underline{v} + \underline{w}$

12) <u>Circle 1</u>

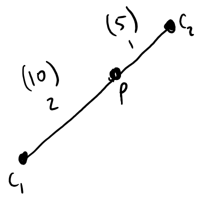
$C_1(-2,-4)$ $r = 10$

<u>Circle 2</u>

$C_2(10,5)$ $r = \sqrt{(-10)^2 + (-5)^2 - 100}$

$r = \sqrt{25}$

$r = 5$



$$P = \frac{2}{3}C_2 + \frac{1}{3}C_1$$

$$3P = 2C_2 + C_1$$

$$3P = 2\begin{pmatrix}10\\5\end{pmatrix} + \begin{pmatrix}-2\\-4\end{pmatrix}$$

$$3P = \begin{pmatrix}20\\10\end{pmatrix} + \begin{pmatrix}-2\\-4\end{pmatrix}$$

$$3P = \begin{pmatrix}18\\6\end{pmatrix}$$

$$P = \begin{pmatrix}6\\2\end{pmatrix} \qquad \underline{\underline{P(6,2)}}$$