

N5

Prelim Examination 2017 / 18

MATHEMATICS
National Qualifications - National 5
Paper 1 (Non Calculator)
Testing EF and REL

Time allowed - 1 hour 15 minutes

Fill in these boxes and read carefully what is printed below

Full name of centre

Town

Forename(s)

Surname

Date of birth

Day Month Year

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Candidate number

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Seat number

Total marks - 50

1. You may NOT use a calculator.
2. Use **blue** or **black** ink. Pencil may be used for graphs and diagrams only.
3. Write your working and answers in the spaces provided. Additional space for answers is provided at the end of the booklet. If you use this space, write clearly the number of the question you are attempting.
4. Square ruled paper is provided.
5. Full credit will be given only where the solution contains appropriate working.
6. State the units for your answer where appropriate.
7. Before leaving the examination room you must give up this booklet to the invigilator. If you do not, you may lose all the marks for this paper.

FORMULAE LIST

The roots of $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$ or $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

Area of a triangle: $\text{Area} = \frac{1}{2} ab \sin C$

Volume of a sphere: $\text{Volume} = \frac{4}{3} \pi r^3$

Volume of a cone: $\text{Volume} = \frac{1}{3} \pi r^2 h$

Volume of a Pyramid: $\text{Volume} = \frac{1}{3} Ah$

Standard deviation: $s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}} = \sqrt{\frac{\sum x^2 - (\sum x)^2 / n}{n-1}}$, where n is the sample size.

4. Change the subject of this formula to V .

$$r = \sqrt{\frac{3V}{\pi h}} \quad 3$$

5. (a) Simplify $\sqrt{50} + \sqrt{8} - 4\sqrt{2}$ 3

- (b) Express with a rational denominator in its simplest form

$$\frac{3}{\sqrt{18}} \quad 3$$

6. A parabola has equation of the form $y = (x+a)^2 + b$.

It has its minimum turning point at (3, 5).

(a) State the equation of the parabola.

2

(b) Determine the point at which this graph crosses the y – axis.

2

(c) What is the equation of the axis of symmetry of the parabola?

1

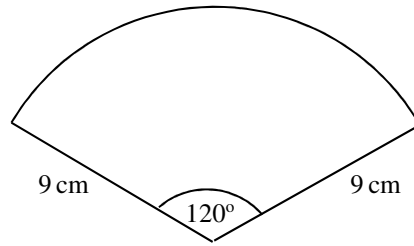
7. Solve the equation $\frac{2x}{5} - \frac{x}{4} = 18$.

3

8. Calculate the perimeter of this sector of the circle which has radius 9 cm and angle at the centre 120° . [Use $\pi = 3.14$]

Marks

3



9. Determine the nature of the roots of the function $2x^2 - 8x + 8 = 0$

2

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10. Write as a single fraction in its simplest form:

3

$$\frac{5}{x-6} - \frac{3}{x+3}; \quad x \neq 6; \quad x \neq -3$$

11. Solve **algebraically** the inequality

$$3x + 5 \leq 7x - 19.$$

3

12 A number pattern is shown below.

Marks

$$1^3 + 1 = (1 + 1)(1^2 - 1 + 1)$$

$$2^3 + 1 = (2 + 1)(2^2 - 2 + 1)$$

$$3^3 + 1 = (3 + 1)(3^2 - 3 + 1)$$

(a) Write down a similar expression for $7^3 + 1$

1

(b) Write down an expression for $n^3 + 1$

1

(c) Find an expression for $8p^3 + 1$

2

- 13 The intensity of light, I , emerging after passing through a liquid with concentration, c , is given by the equation

$$I = \frac{20}{2^c} \quad c \geq 0$$

- (a) Find the intensity of light when the concentration is 3. 1

- (b) Find the concentration of the liquid when the intensity is 10 2

- (c) What is the maximum possible intensity? 2

Qu	Give one mark for each •	Illustrations for awarding mark
1a	Ans: $3(a + 4)(a - 4)$ 2 marks 1. Common Factor	• $3(a^2 - 16)$ • $3(a + 4)(a - 4)$
1b	Ans: $(x - 4)(x + 3)$ 3 marks 1. Finds one factor 2. Finds second factor 3. Simplifies	• $(x - 4)$ • $(x + 3)$ • $\frac{5}{x+3}$
2	ans: $3y + 2x = 24$ [or equi.] 4 marks • ¹ knows how to find gradient • ² subs into equation • ³ Rearrange • ⁴ Answer	• ¹ $m = \frac{10 - 6}{-3 - 3} = \frac{4}{-6} = -\frac{2}{3}$ • ² $y - 6 = -\frac{2}{3}(x - 3)$ or $y - 10 = -\frac{2}{3}(x + 3)$ • ³ $3y - 18 = -2x + 6$ or $3y - 30 = -2x - 6$ • ⁴ $3y + 2x = 24$ or $y = -\frac{2}{3}x + 8$
3a	ans: $x^{\frac{5}{2}} + x^{\frac{9}{2}}$ 2 marks • ¹ first term correct • ² second term correct	• ¹ $x^{\frac{5}{2}} \dots\dots$ • ² $\dots\dots + x^{\frac{9}{2}}$
b	ans: 125 2 marks • ¹ knows meaning of fractional index • ² processes answer	• ¹ $\sqrt{25^3}$ • ² 125
4	ans: $V = \frac{\pi r^2 h}{3}$ 3 marks • ¹ removes square root • ² removes fraction • ³ answer	• ¹ $r^2 = \frac{3V}{\pi h}$ • ² $3V = \pi r^2 h$ • ³ $V = \frac{\pi r^2 h}{3}$
5a	ans: $3\sqrt{2}$ 3 marks • ¹ simplifies $\sqrt{50}$ • ² simplifies $\sqrt{8}$ • ³ answer	• ¹ $5\sqrt{2}$ • ² $2\sqrt{2}$ • ³ $3\sqrt{2}$
b	ans: $\frac{\sqrt{2}}{2}$ 3 marks • ¹ simplifies $\sqrt{18}$ • ² knows to multiply by $\frac{\sqrt{2}}{\sqrt{2}}$ • ³ final answer	• ¹ $\frac{3}{3\sqrt{2}}$ • ² $\frac{3}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ • ³ $\frac{3\sqrt{2}}{6} = \frac{\sqrt{2}}{2}$

Qu	Give one mark for each •	Illustrations for awarding mark
6a	ans: $y = (x-3)^2 + 5$ 2 marks <ul style="list-style-type: none"> •¹ subs correctly for a •² subs correctly for b 	<ul style="list-style-type: none"> •¹ $y = (x-3)^2 \dots$ •² $y = (x-3)^2 + 5$
b	ans: (0, 14) 2 marks <ul style="list-style-type: none"> •¹ knows to sub $x = 0$ •² states point 	<ul style="list-style-type: none"> •¹ $y = (0-3)^2 + 5$ •² (0, 14) [must be coordinate point]
c	ans: $x = 3$ 1 mark <ul style="list-style-type: none"> •¹ answer 	<ul style="list-style-type: none"> •¹ $x = 3$
7	ans: $x = 120$ 3 marks <ul style="list-style-type: none"> •¹ subtracts fractions •² multiplies through by 20 •³ solves for x 	<ul style="list-style-type: none"> •¹ $\frac{3x}{20} = 18$ •² $3x = 360$ •³ $x = 120$
8	ans: 36.84cm 3 marks <ul style="list-style-type: none"> •¹ subs correct values into formula •² starts to evaluate •³ completes evaluation and adds radii 	<ul style="list-style-type: none"> •¹ $\frac{120}{360} \times 3 \cdot 14 \times 18$ •² $\frac{1}{3} \times 3 \cdot 14 \times 18 = 3 \cdot 14 \times 6$ •³ $18 \cdot 84 + 18 = 36 \cdot 84 \text{ cm}$ [units required]
9	ans: $b^2 - 4ac = 0$ 2marks <i>equal real roots</i> <ul style="list-style-type: none"> •¹ substitutes values •² knows condition for equal roots 	<ul style="list-style-type: none"> • $(-8)^2 - 4 \times 2 \times 8 = 0$ • $b^2 - 4ac = 0$ for equal real roots
10	ans: $\frac{2x+33}{(x+3)(x-6)}$ 3 marks <ul style="list-style-type: none"> •¹ correct denominator •² correct numerator •³ simplifies numerator 	<ul style="list-style-type: none"> •¹ $(x+3)(x-6)$ •² $5(x+3) - 3(x-6)$ •³ $2x+33$
11	ans: $x \geq 6$ 3 marks <ul style="list-style-type: none"> •¹ swaps sides and changes sign •² subtracts $3x$ from both sides •³ solves 	<ul style="list-style-type: none"> •¹ $7x - 19 \geq 3x + 5$ •² $4x - 19 \geq 5$ •³ $x \geq 6$
12	(a) Ans $(7+1)(7^2 - 7 + 1)$ 1mark (b) $(n+1)(n^2 - 7 + 1)$ 1mark $(2p+1)((2p)^2 - 2p + 1)$ 2marks	<ul style="list-style-type: none"> • 2p • Answer

13 (a)	$\frac{5}{2}$	1 mark	
(b)	<ul style="list-style-type: none"> • Substitution • $C = 1$ 	2 marks	
(c)	$I = 20$ when $C = 0$	2 marks	<ul style="list-style-type: none"> • $C = 0$ • $I = 20$
			Total 50 marks



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All questions should be attempted

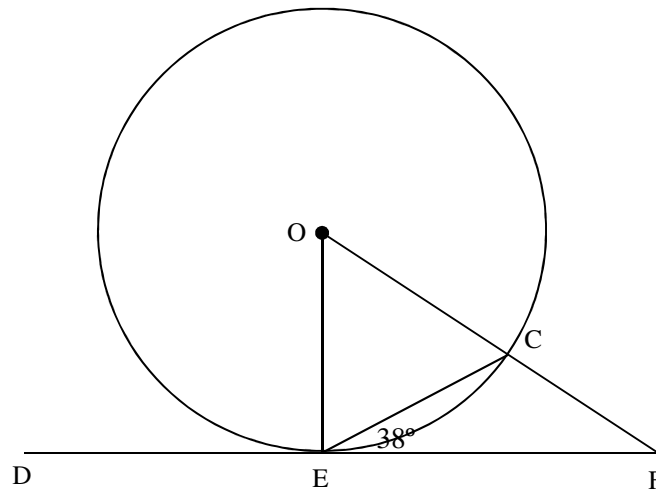
Do not
write in
this
margin.

Marks

1. The line DF is a tangent to the circle centre O shown below. E is the point of contact of the tangent.

Given that angle CEF is 38° , calculate the size of angle CFE.

3

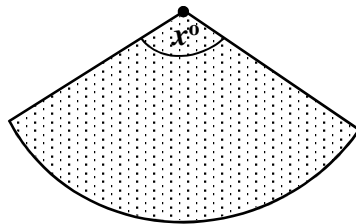


2. Write $x^2 - 10x + 1$ in the form $(x+a)^2 + b$.

2

3. An international space station travels around 2.3×10^6 km each time it completes an orbit the Earth.
It completes 0.65 orbits per hour.
Calculate how far it will travel in 2017.
Give your answer in scientific notation correct to three significant figures. 3

4. A sensor on a security system covers a horizontal area in the shape of a sector of a circle of radius 10 m.



The area covered by the sensor is 96 square metres.

Find the angle, x° , at the centre of the sector. 4

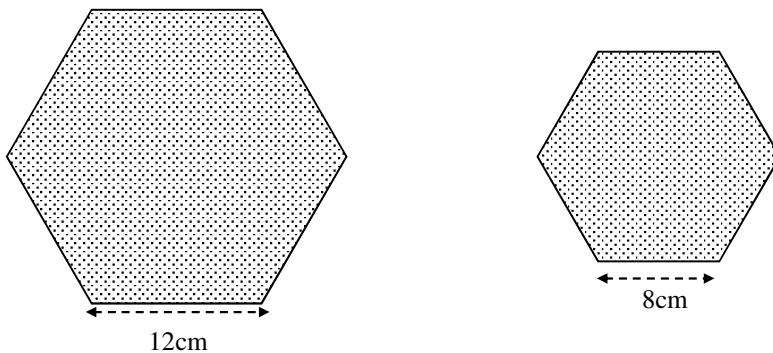
5. Solve the quadratic equation

$$3x^2 - 9x + 2 = 0$$

Give your answers correct to 1 decimal place.

3

6. These two hexagons are mathematically similar.



If the area of the smaller hexagon is 144cm^2 , find the area of the larger one?

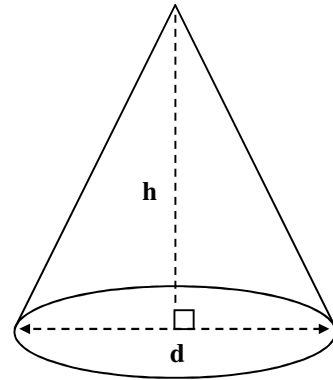
3

7. Mr Smith and Mrs Curran both shop at the same store.
- (a) Mr Smith bought 3 loaves and 2 packets of butter. The total cost was £4.73. **1**
Let x pounds be the cost of a loaf and y pounds be the cost of a packet of butter.
Write down an equation in x and y which satisfies the above condition.
- (b) Mrs Curran bought 5 loaves and 3 packets of butter. The total cost was £7.52. **1**
Write down a second equation in x and y which satisfies this condition.
- (c) Calculate the cost of one loaf of bread and the cost of one packet of butter. **4**
8. Multiply the brackets and simplify $(3x - 2)(4x^2 - 5x + 1)$ **3**

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9. (a) Find the volume of this cone which has diameter 18cm and height 20cm.

2

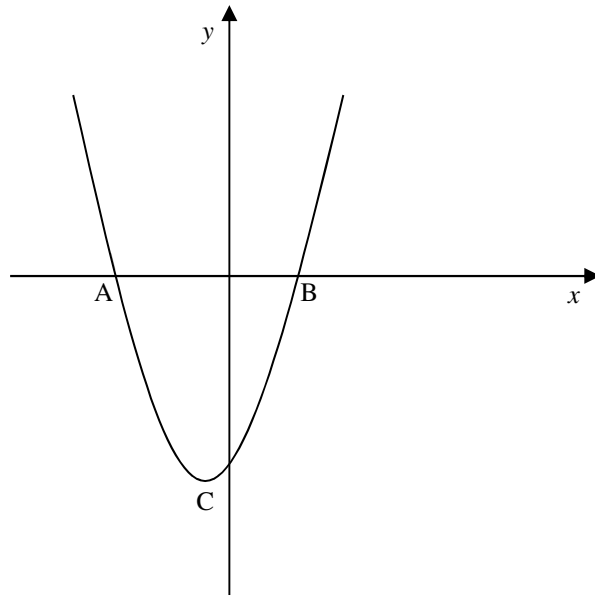


- (b) If the cone is re-modelled into a sphere which has the same volume, find the diameter of the sphere.

3

A vertical rectangular box for writing the answer to question 9(b).

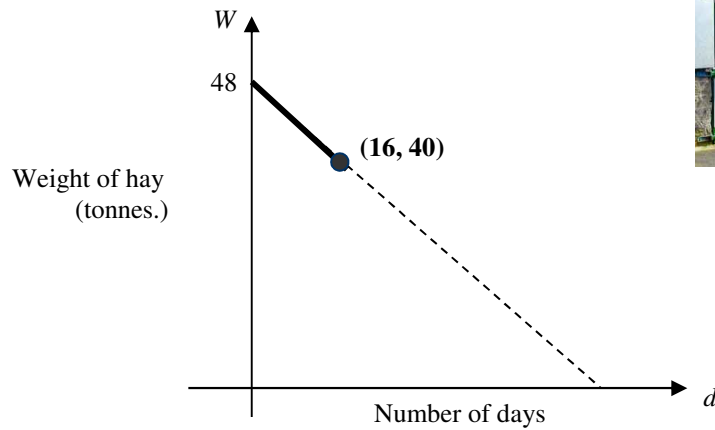
11. Paul was asked to make a sketch of the graph of $y = x^2 + 6x - 16$.
This is the graph that he drew.



Find the numbers that should be placed at the points A, B and the coordinates of point, C, the turning point of the parabola.

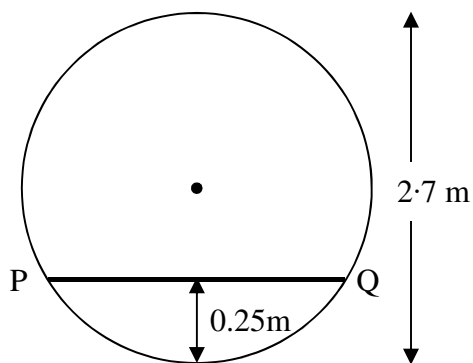
5

12. The local riding stables buy in 48 tonnes of hay to feed the horses during the winter season, which lasts for 93 days. After 16 days they have 40 tonnes of hay left. The graph below illustrates the situation.



- (a) Find the equation of the line shown above in terms of W and d . 3
- (b) If the horses continue to consume the hay at this rate, will it last to the end of the winter season? 3

13.



A circular service tunnel of diameter 2.7 metres has a metal platform, PQ, whose centre is 0.25 metres from the bottom of the tunnel.

Calculate the width of the platform.

4

14. A is the point (a^2, a)
T is the point (t^2, t) $a \neq t$

Find the gradient of the line AT
Give your answer in its simplest form.

3

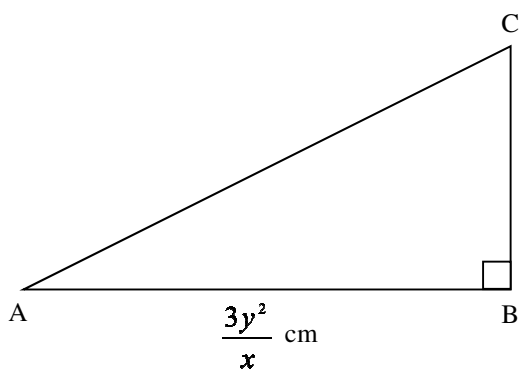
15. The number of diagonals, d , in a polygon with n sides is given by the formula:

$$d = \frac{n(n-3)}{2}$$

A polygon has 20 diagonals
How many sides does it have?

4

- 16.



In triangle ABC, AB is $\frac{3y^2}{x}$ cm and
its area is $6y$ cm².

Calculate the length of BC, expressing
your answer in its simplest form.

3

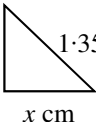
- 10** In 2013, a house was valued at £90 000 and the contents were valued at £60 000.
The value of the house appreciates by 5% each year.
The value of the contents depreciates by 8% each year.
What will the value of the house and contents be in 2017?

3

ADDITIONAL SPACE FOR ANSWERS

Qu	Give one mark for each •	Illustrations for awarding mark
1	ans : 14° 3 marks <ul style="list-style-type: none"> •¹ knows OEF is right and finds OEC •² knows OCE = OEC and finds ECF •³ finds required angle 	<ul style="list-style-type: none"> •¹ angle OEC = 52° •² angle OCE = 52° and angle EFC = 128° •³ EFC = $180 - (38 + 128) = 14^\circ$ [stated explicitly]
2	ans : $(x - 5)^2 - 24$ 2 marks <ul style="list-style-type: none"> •¹ starts process •² completes process 	<ul style="list-style-type: none"> •¹ $(x - 5)^2 \dots\dots$ •² $(x - 5)^2 - 25 + 1$
3	ans: 1.31×10^{10} 3 marks <ul style="list-style-type: none"> •¹ calculation •² answer •³ answer correctly rounded 	<ul style="list-style-type: none"> •¹ $2.3 \times 10^6 \times 0.65 \times 24 \times 365$ •² 1.30962×10^{10} •³ 1.31×10^{10}
4	ans : 110° 4 marks <ul style="list-style-type: none"> •¹ finds area of circle •² equates two fractions •³ cross multiplies •⁴ processes answer 	<ul style="list-style-type: none"> •¹ $A = \pi \times 10^2 = 100\pi$ [or equivalent] •² $\frac{x}{360} = \frac{96}{100\pi}$ •³ $x = \frac{96 \times 360}{100\pi}$ •⁴ 110°
5	ans : $2.8, 0.2$ 3 marks <ul style="list-style-type: none"> •¹ substitutes into quadratic formula correctly •² calculates $b^2 - 4ac$ •³ states both roots correctly rounded 	<ul style="list-style-type: none"> •¹ $\frac{9 \pm \sqrt{(-9)^2 - 4 \times 3 \times 2}}{2 \times 3}$ •² 57 •³ 2.8, 0.2

Qu	Give one mark for each •	Illustrations for awarding mark
6	ans: 324cm^2 3 marks <ul style="list-style-type: none"> •¹ finds linear scale factor for enlargement •² finds area scale factor •³ multiplies by ASF to answer 	<ul style="list-style-type: none"> •¹ $\frac{12}{8} = \frac{3}{2}$ •² $(\frac{3}{2})^2$ •³ $(\frac{3}{2})^2 \times 144 = 324 \text{ cm}^2$
7	ans: £1.94 5 marks <ul style="list-style-type: none"> •¹ Finds first equation •² finds second equation •³ scales •⁴ follow a valid strategy to produce values for x and y •⁵ Correct values for x and y •⁵ communicates answers 	<ul style="list-style-type: none"> •¹ $3x + 2y = 4.73$ •² $5x + 3y = 7.52$ •³ •⁴ values for x and y •⁵ $x = £1.09$ $y = £0.85$ •⁶ Loaf of Bread = £1.09 and Butter = £0.85
8	ans: $12x^3 - 23x^2 + 13x - 2$ 3 marks <ul style="list-style-type: none"> •¹ any three terms correct •² further three terms correct •³ simplifies 	<ul style="list-style-type: none"> •¹ $12x^3 - 15x^2 + 3x \dots$ •² $\dots - 8x^2 + 10x - 2$ •³ $12x^3 - 23x^2 + 13x - 2$
9a	ans: 1695.6cm^3 2 marks <ul style="list-style-type: none"> •¹ subs values into correct formula •² finds volume of cone 	<ul style="list-style-type: none"> •¹ $V_{\text{cone}} = \frac{1}{3} \times \pi \times 9^2 \times 20$ •² 1696.5cm^3
b	ans: 14.8cm 3 marks <ul style="list-style-type: none"> •¹ equates above to volume of sphere •² starts to find r •³ evaluates and states diameter 	<ul style="list-style-type: none"> •¹ $1695.6 = \frac{4}{3} \times \pi \times r^3$ •² $r = \sqrt[3]{\frac{1696.5}{\frac{4}{3} \times 3.14}} \text{ cm}^3$ •³ $r = 7.4\text{cm}$ and diameter = 14.8cm
10	Ans £109385.56 3marks Ans £42993.58 Ans £152379.14	<ul style="list-style-type: none"> •¹ $90000 \times 1.05^4 = £109395.56$ •² $60000 \times 0.92^4 = £42983.58$ •³ £152379.14
11	ans : A = -8; B = 2; C(-3,-25) 5 marks <ul style="list-style-type: none"> •¹ equates to 0 •² factorises and solves •³ states numbers at A and B •⁴ subs -3 into equation and evaluates •⁵ states coordinates of C 	<ul style="list-style-type: none"> •¹ $x^2 + 6x - 16 = 0$ •² $(x + 8)(x - 2) = 0$; $x = -8$; 2 •³ $A = -8$; $B = 2$ •⁴ $(-3)^2 + 6(-3) - 16 = -25$ •⁵ $C(-3, -25)$

Qu	Give one mark for each •	Illustrations for awarding mark
12a b	<p>ans: $W = -0.5d + 48$</p> <ul style="list-style-type: none"> •¹ identifies y - intercept •² calculates gradient •³ states equation <p>ans: Yes, 3 days spare</p> <ul style="list-style-type: none"> •¹ correct strategy •² solves equation •³ correct conclusion 	<ul style="list-style-type: none"> •¹ $c = 48$ •² $m = \frac{48 - 40}{0 - 16} = -0.5$ •³ $W = -0.5d + 48$ <ul style="list-style-type: none"> •¹ $-0.5d + 48 = 0$ •² $d = 96$ •³ yes, 3 days to spare
13	<p>ans : 1.57m 4 marks</p> <ul style="list-style-type: none"> •¹ assembles facts in right triangle •² knows to use Pythagoras' •³ uses Pythagoras' correctly •⁴ finds width 	<div style="text-align: right;">  </div> <ul style="list-style-type: none"> •¹ •² $x^2 = 1.35^2 - 1.1^2$ •³ $x = 0.78$ •⁴ $w = 1.57\text{cm}$
14	<p>ans: $m = \frac{1}{t + a}$ 3marks</p> <ul style="list-style-type: none"> •¹ substitutes into formula •² factorise denominator •³ simplifies fraction 	<ul style="list-style-type: none"> •¹ $m = \frac{t - a}{t^2 - a^2}$ •² $(t - a)(t + a)$ •³ $m = \frac{1}{t + a}$
15	<p>ans: $n = 8$ 4marks $n(n-3)/2 = 20$ $n^2 - 3n = 40$ $n^2 - 3n - 40 = 0 (n+5)(n - 8)$ n can't equal -5 so $n = 8$</p>	<p>Substitution Remove brackets Factorise Solve and dismiss -5</p>
16	<p>ans: $\frac{4x}{y}$ 3 marks</p> <ul style="list-style-type: none"> •¹ knows how to find length of BC •² inverts divisor and multiplies •³ answer 	<ul style="list-style-type: none"> •¹ $6y \div \frac{3y^2}{2x}$ •² $6y \times \frac{2x}{3y^2}$ •³ $\frac{4x}{y}$