Dingwall Academy

Mathematics Department

Mathematics N5

UNIT 1

Revision Materials

Expressions and Formulae Skills Builder

Layout and content of the Unit Assessment will be different. This is not meant to be a carbon copy of the Unit Assessment. This booklet is an opportunity to practice all of the essential skills required to pass the Unit Assessment.

Outcomes and assessment standards

Outcome 1

The learner will:
1 Use mathematical operational skills linked to expressions and formulae by:

EF1.1 Applying numerical skills to simplify surds/expressions using the laws of indices
EF1.2 Applying algebraic skills to manipulate expressions
EF1.3 Applying algebraic skills to algebraic fractions
EF1.4 Applying geometric skills linked to the use of formulae
2 Use mathematical reasoning skills linked to expressions and formulae by:

EF2.1 Interpreting a situation where mathematics can be used and identifying a valid strategy
EF2.2 Explaining a solution and/or relating it to context

FORMULAE SHEET

The roots of $\quad ax^2 + bx + c = 0 \quad are \ x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sine Rule: $\quad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine Rule: $\quad a^2 = b^2 + c^2 - 2bc \cos A \quad or \quad \cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

Area of a triangle $\quad A = \dfrac{1}{2} ab \sin C$

Volume of a sphere $\quad V = \dfrac{4}{3} \pi r^3$

Volume of a cone $\quad V = \dfrac{1}{3} \pi r^2 h$

Volume of a Pyramid $\quad V = \dfrac{1}{3} Ah$

Standard Deviation $\quad s = \sqrt{\dfrac{\sum(x - \bar{x})^2}{n - 1}} = \sqrt{\dfrac{\sum x^2 - (\sum x)^2/n}{n - 1}} \quad where \ n \ is \ the \ sample \ size$

SURDS

1 Use mathematical operational skills linked to expressions and formulae by:
EF1.1 Applying numerical skills to simplify surds/expressions using the laws of indices

1. Copy and complete the following

    a. $\sqrt{20} = \sqrt{(4 \times \ldots)} = \sqrt{\ldots} \times \sqrt{\ldots} = \ldots \sqrt{\ldots}$
    b. $\sqrt{75} = \sqrt{(\ldots \times 3)} = \sqrt{\ldots} \times \sqrt{\ldots} = \ldots \sqrt{\ldots}$
    c. $\sqrt{96} = \sqrt{(\ldots \times \ldots)} = \sqrt{\ldots} \times \sqrt{\ldots} = \ldots \sqrt{\ldots}$
    d. $\sqrt{128} = \sqrt{(\ldots \times \ldots)} = \sqrt{\ldots} \times \sqrt{\ldots} = \ldots \sqrt{\ldots}$
    e. $\sqrt{175} = \sqrt{(\ldots \times \ldots)} = \sqrt{\ldots} \times \sqrt{\ldots} = \ldots \sqrt{\ldots}$
    f. $\sqrt{300} = \sqrt{(\ldots \times \ldots)} = \sqrt{\ldots} \times \sqrt{\ldots} = \ldots \sqrt{\ldots}$
    g. $\sqrt{45} = \sqrt{(\ldots \times \ldots)} = \sqrt{\ldots} \times \sqrt{\ldots} = \ldots \sqrt{\ldots}$

2. Simplify the following surds

a)	$\sqrt{18}$	**b)**	$\sqrt{8}$	**c)**	$\sqrt{24}$	**d)**	$\sqrt{27}$	**e)**	$\sqrt{200}$
f)	$\sqrt{32}$	**g)**	$\sqrt{72}$	**h)**	$\sqrt{147}$	**i)**	$\sqrt{128}$	**j)**	$\sqrt{45}$
k)	$\sqrt{28}$	**l)**	$\sqrt{405}$	**m)**	$\sqrt{180}$	**n)**	$\sqrt{112}$	**o)**	$\sqrt{245}$

3. Simplify:

    a) $5\sqrt{2} + 3\sqrt{2}$ **b)** $9\sqrt{11} - 3\sqrt{11}$

    c) $8\sqrt{13} + 2\sqrt{13} - 9\sqrt{13}$ **d)** $\sqrt{162} - 6\sqrt{2}$

    e) $6\sqrt{5} - \sqrt{125}$ **f)** $\sqrt{96} - \sqrt{24}$

    g) $\sqrt{72} - \sqrt{32}$ **h)** $\sqrt{24} + \sqrt{54}$

    i) $\sqrt{12} + \sqrt{48}$ **j)** $\sqrt{20} + \sqrt{80}$

    k) $\sqrt{44} + \sqrt{99}$ **l)** $\sqrt{40} + \sqrt{90}$

    m) $\sqrt{242} - \sqrt{28} + \sqrt{175}$ **n)** $\sqrt{50} + \sqrt{75} + \sqrt{98} + \sqrt{147}$

    o) $\sqrt{300} + \sqrt{72} - \sqrt{108} + \sqrt{242}$

4. Simplify

a) $\sqrt{3} \times \sqrt{3}$ **b)** $\sqrt{7} \times \sqrt{7}$ **c)** $2\sqrt{10} \times \sqrt{10}$ **d)** $5\sqrt{11} \times 2\sqrt{11}$

e) $3\sqrt{5} \times 6\sqrt{5}$ **f)** $\sqrt{8} \times \sqrt{2}$ **g)** $\sqrt{5} \times \sqrt{20}$ **h)** $\sqrt{20} \times \sqrt{8}$

5. Simplify the following and leave your answer in surd form where applicable.

a) $3\sqrt{5} \times 2\sqrt{5}$ **b)** $4\sqrt{6} \times 3\sqrt{8}$ **c)** $2\sqrt{5} \times 3\sqrt{10}$

d) $2\sqrt{8} \times 3\sqrt{10}$ **e)** $5\sqrt{8} \times 4\sqrt{2}$ **f)** $3\sqrt{3} \times 4\sqrt{6}$

6. Simplify the following

a) $\dfrac{\sqrt{20}}{\sqrt{5}}$ **b)** $\dfrac{\sqrt{24}}{\sqrt{3}}$ **c)** $\dfrac{\sqrt{80}}{\sqrt{20}}$ **d)** $\dfrac{\sqrt{72}}{\sqrt{8}}$ **e)** $\dfrac{\sqrt{75}}{\sqrt{3}}$

f) $\dfrac{\sqrt{48}}{\sqrt{6}}$ **g)** $\dfrac{\sqrt{50}}{\sqrt{2}}$ **h)** $\dfrac{\sqrt{84}}{\sqrt{7}}$ **i)** $\dfrac{\sqrt{60}}{\sqrt{3}}$ **j)** $\dfrac{\sqrt{90}}{\sqrt{5}}$

k) $\dfrac{\sqrt{7}}{\sqrt{56}}$ **l)** $\dfrac{\sqrt{50}}{\sqrt{18}}$ **m)** $\dfrac{\sqrt{5}}{\sqrt{200}}$ **n)** $\dfrac{\sqrt{6}}{\sqrt{120}}$ **o)** $\dfrac{\sqrt{10}}{\sqrt{500}}$

7. Simplify the following

a) $\sqrt{\dfrac{8}{18}}$ **b)** $\sqrt{\dfrac{20}{45}}$ **c)** $\sqrt{\dfrac{24}{54}}$ **d)** $\sqrt{\dfrac{54}{96}}$ **e)** $\sqrt{\dfrac{28}{63}}$

f) $\sqrt{\dfrac{108}{147}}$ **g)** $\sqrt{\dfrac{162}{98}}$ **h)** $\sqrt{\dfrac{192}{75}}$ **i)** $\sqrt{\dfrac{112}{63}}$ **j)** $\sqrt{\dfrac{20}{125}}$

k) $\sqrt{\dfrac{60}{375}}$ **l)** $\sqrt{\dfrac{117}{52}}$ **m)** $\sqrt{\dfrac{44}{99}}$ **n)** $\sqrt{\dfrac{50}{98}}$ **o)** $\sqrt{\dfrac{24}{96}}$

8. By converting to fractions, calculate the following:

a) $\sqrt{0.16}$ **b)** $\sqrt{0.36}$ **c)** $\sqrt{0.64}$ **d)** $\sqrt{0.04}$

e) $\sqrt{0.0025}$ **f)** $\sqrt{0.0081}$ **g)** $\sqrt{0.000049}$ **h)** $\sqrt{0.0000001}$

INDICES

1 Use mathematical operational skills linked to expressions and formulae by:
EF1.1 Applying numerical skills to simplify surds/expressions using the laws of indices

9. Simplify

a) $x \times x^2$ **b)** $t^3 \times t^2$ **c)** $b^4 \times b^{-1}$ **d)** $a \times a^3$

e) $g^4 \times g^9$ **f)** $d^2 \times d^2$ **g)** $v^{13} \times v^3$ **h)** $h^3 \times h^{-3}$

i) $m \times m^2 \times m^3$ **j)** $d^2 \times d^5 \times d^8$ **k)** $z^3 \times z \times z^2$ **l)** $p^7 \times p^3 \times p^5$

m) $g \times g^4 \times g \times g^7$ **n)** $j^2 \times j \times j^4 \times j^{-1}$ **o)** $h^{22} \times h^{-13} \times h$ **p)** $y^{1/2} \times y \times y^2 \times y^5$

q) $5^{13} \times 5^4$ **r)** $13^{10} \times 13^{11}$ **s)** $8^4 \times 8^{15}$ **t)** $4^3 \times 4^7 \times 4^{10}$

10. Simplify

a) $3x \times 2x$ **b)** $2t^2 \times 5t$ **c)** $3b^2 \times b^4$ **d)** $4a^7 \times a^3$

e) $13g^2 \times 2g^3$ **f)** $8d^2 \times 8d^2$ **g)** $4v^2 \times -3v^5$ **h)** $-2h^3 \times 4h^2$

i) $-m \times 2m^2$ **j)** $2d^2 \times d^2 \times 4d^7$ **k)** $13z^4 \times 12z$ **l)** $-7p^8 \times -8p^7$

m) $4g \times 3g^{-1}$ **n)** $18j^6 \times 2j^{-9}$ **o)** $-3h^5 \times 4h^{-13}$ **p)** $9y^{1/2} \times 2y$

q) $2f^3 \times 5f^2 \times 6$ **r)** $2^2 \times 5x^3$ **s)** $3^4 \times -2m^{15}$ **t)** $4^3 \times 2x^7$

11. Simplify

a) $\dfrac{x^6}{x}$ **b)** $\dfrac{x^6}{x^3}$ **c)** $\dfrac{x^{10}}{x^4}$ **d)** $\dfrac{x^{12}}{x^7}$

e) $\dfrac{x^{13}}{x^2}$ **f)** $\dfrac{a^{11}}{a^2}$ **g)** $\dfrac{b^8}{b^3}$ **h)** $\dfrac{z^9}{z^5}$

i) $y^5 \div y^2$ **j)** $z^{15} \div z^{12}$ **k)** $m^7 \div m^6$ **l)** $t^4 \div t^4$

m) $c^{23} \div c^{19}$ **n)** $h^9 \div h^{10}$ **o)** $f^{11} \div f^3$ **p)** $q^{20} \div q^5$

q) $\dfrac{6k^4}{2k^3}$ **r)** $\dfrac{15r^7}{3r^5}$ **s)** $\dfrac{63n^8}{7n^5}$ **t)** $\dfrac{18x^3}{12x^{1/2}}$

u) $\dfrac{0.6^9}{0.6^4}$ **v)** $\dfrac{1.2^7}{1.2^2}$ **w)** $\dfrac{\left(\frac{2}{3}\right)^5}{\left(\frac{2}{3}\right)^3}$ **x)** $\dfrac{(3x)^4}{3x}$

12. Simplify the following

a) $\dfrac{x^4 \times x^2}{x^3}$ **b)** $\dfrac{t^6 \times t^5}{t^4}$ **c)** $\dfrac{b^4 \times b^5}{b^8}$ **d)** $\dfrac{4a \times a^9}{2a^5}$

e) $\dfrac{3g^5 \times 6g^4}{4g^5}$ **f)** $\dfrac{3d^2 \times 17d^4}{4d^3}$ **g)** $\dfrac{6v^7 \times 3v^3}{4v^9}$ **h)** $\dfrac{7h^3 \times 8h^6}{21h^5}$

i) $\dfrac{m \times 4m^2 \times 3m^3}{7m^4}$ **j)** $\dfrac{d^2 \times 15d^5 \times 2d^8}{6d^9}$ **k)** $\dfrac{3z \times (-z)^3 \times z^2}{2z^4}$ **l)** $\dfrac{-p^7 \times -p^3 \times (-p)^5}{-3p^8}$

13. Simplify the following

a $\dfrac{a^3 \times a^{\frac{3}{2}}}{a^4}$ **b** $\dfrac{b^{\frac{1}{3}} \times b^4}{b^{\frac{2}{3}}}$

c $\dfrac{c^{\frac{5}{2}} \times c^{-\frac{1}{2}}}{c^3}$ **d** $\dfrac{d^3 \times d^6}{d^{\frac{7}{3}}}$

e $\dfrac{E^{-2} \times E^{\frac{3}{2}}}{E^{\frac{1}{2}}}$ **f** $\dfrac{f^{-\frac{2}{3}} \times f^3}{f^{\frac{4}{3}}}$

g $\dfrac{g^{\frac{1}{2}} \times g^{\frac{1}{2}}}{g^{-\frac{1}{2}}}$ **h** $\dfrac{h^{\frac{1}{2}} \times h^{\frac{1}{3}}}{h^{\frac{1}{4}}}$

SCIENTIFIC NOTATION

1 Use mathematical operational skills linked to expressions and formulae by:
EF1.1 Applying numerical skills to simplify surds/expressions using the laws of indices

14. Write each of the following numbers in scientific notation.

a) 300 000 000 000 **b)** 80 000 000 **c)** 700 000 000 **d)** 2 000 000 000

e) 42 000 000 **f)** 21 000 000 000 **g)** 3 700 000 000 **h)** 630

15. Write each of the following numbers in scientific notation.

a)   0.125   **b)**   0.0024   **c)**   0.00000012   **d)**   0.00000249

e)   0.00412   **f)**   0.00049   **g)**   0.000000307   **h)**   0.0020107

16. Write the following in normal form

a) 2.3×10^3 **b)** 1.04×10^6 **c)** 1.56×10^5 **d)** 9.03×10^7

e) 4.1×10^9 **f)** 6.0001×10^2 **g)** 3.56×10^6 **h)** 7.12×10^{11}

17. Write the following in normal form

a) 3.4×10^{-2} **b)** 6.021×10^{-6} **c)** 8.201×10^{-5} **d)** 6.549×10^{-3}

e) 1.3×10^{-7} **f)** 5.001×10^{-9} **g)** 7.99×10^{-3} **h)** 4.09×10^{-1}

18. Calculate the following

a $1.254 \times 10^3 + 6.2 \times 10^2$ **b** $3.6 \times 10^7 + 8.3 \times 10^5$

c $9.34 \times 10^4 + 6.25 \times 10^5$ **d** $8.1 \times 10^4 + 6.4 \times 10^4$

e $5.74 \times 10^6 - 8.2 \times 10^4$ **f** $9.13 \times 10^4 - 5.64 \times 10^3$

g $8.35 \times 10^6 - 6.14 \times 10^6$ **h** $4.99 \times 10^7 - 1.0 \times 10^6$

i $1.01 \times 10^9 - 1.01 \times 10^7$ **j** $6.35 \times 10^2 - 5.36 \times 10^2$

k $5.1 \times 10^8 \times 2.0 \times 10^{-2}$ **l** $3.4 \times 10^6 \times 1.1 \times 10^5$

m $6.5 \times 10^7 \times 1.2 \times 10^{-7}$ **n** $2.4 \times 10^6 \div 8.0 \times 10$

o $5.2 \times 10^8 \div 1.3 \times 10^7$ **p** $4.9 \times 10^5 \div 7.0 \times 10^{-2}$

1 Use mathematical operational skills linked to expressions and formulae by:
EF1.1 Applying numerical skills to simplify surds/expressions using the laws of indices
2 Use mathematical reasoning skills linked to expressions and formulae by:
EF2.1 Interpreting a situation where mathematics can be used and identifying a valid strategy
EF2.2 Explaining a solution and/or relating it to context

19. Use the information in the table below to answer the following questions.

	Mass (kg)	Average Distance from the sun (km)
Mercury	3.3×10^{23}	5.79×10^{7}
Venus	4.87×10^{24}	1.082×10^{8}
Earth	5.97×10^{24}	1.496×10^{8}
The Moon	7.3×10^{22}	
Mars	6.42×10^{23}	2.066×10^{8}
Jupiter	1.898×10^{27}	7.786×10^{8}
Saturn	5.68×10^{26}	1.4335×10^{9}
Uranus	8.68×10^{25}	2.8725×10^{9}
Neptune	1.02×10^{26}	4.4951×10^{9}
Pluto	1.46×10^{22}	5.9064×10^{9}
The Sun	1.989×10^{30}	
Speed of light	$1.07925 \times 10^{9} \ km/h$	

a. How many times bigger is The Sun compared to Earth?

b. How many times bigger is the average distance from The Sun of Neptune compared to Mercury?

c. How many times bigger is The Sun compared to the Jupiter?

d. How many times bigger is the Earth compared to the Mars?

e. How many times bigger is the average distance from The Sun of Saturn compared to Earth?

f. Using the speed of light provided, how long would it take for light to cover the average distance of each of the planets from The Sun?

g. Pluto's distance from the sun has a minimum of $4.4368 \times 10^{9} \ km$ and a maximum distance of $7.3759 \times 10^{9} \ km$. Calculate how much longer light takes to cover the distance when it is furthest away from the sun compared to when it is closest.

BRACKETS AND EQUATIONS

1 Use mathematical operational skills linked to expressions and formulae by:
EF1.2 Applying algebraic skills to manipulate expressions

20. Expand and simplify where appropriate:

a) $7x(x - 2y)$ **b)** $4x(x^2 + 2x - 7)$ **c)** $-ax^2(3 - x)$ **d)** $x - 3(4 - x)$

e) $(x + 4)^2$ **f)** $(2x - 3)^2$ **g)** $(4 - x)^2$ **h)** $(x^2 + 1)^2$

i) $(x + 1)(x - 3)$ **j)** $(2x + 3)(x - 7)$ **k)** $(3x - 4)(x^2 + 1)$ **l)** $(3 - x)(x + 4)$

m) $(x - 3)(x + 3)$ **n)** $(2x - 5)(2x + 5)$ **o)** $(7 - x)(7 + x)$ **p)** $3(x + 1)(x - 1)$

q) $(x + 1)(x^2 + x + 2)$ **r)** $(x - 1)(x^2 + x - 6)$ **s)** $(x^2 + 3x + 2)(x + 2)$ **t)** $2(x + 4) - (x - 3)$

u) $3(x + 1) + 5(2 - x)$ **v)** $3 - (x + 1)^2$ **w)** $(x - 1)^2 - (6 - x)$ **x)** $3x^2 - (x - 3)(2 - x)$

21. Factorise the following expressions fully.

a) $x^2 - 4$ **b)** $x^2 - 16$ **c)** $x^2 - 9$ **d)** $x^2 - 25$

e) $4 - x^2$ **f)** $9 - x^2$ **g)** $49 - x^2$ **h)** $3x^2 - 3$

i) $2x^2 - 18$ **j)** $4x^2 - 25$ **k)** $25x^2 - 1$ **l)** $1 - 16x^2$

m) $7x^2 - 28$ **n)** $9x^2 - 121$ **o)** $\frac{1}{2}x^2 - 2$ **p)** $\frac{1}{4}x^2 - 1$

22. Factorise the following expressions fully.

a. $x^2 - 2x + 1$

b. $z^2 - 6z + 9$

c. $x^2 + 7x + 10$

d. $x^2 - 9x + 18$

e. $x^2 - 11x + 24$

f. $x^2 + 3x - 4$

g. $x^2 + x - 6$

h. $x^2 - x - 6$

i. $x^2 - 5x - 6$

j. $x^2 - 4x - 12$

k. $x^2 - 2x - 15$

l. $x^2 - 10x - 11$

m. $x^2 + x - 30$

n. $x^2 + 3x - 28$

o. $x^2 - 10x + 16$

p. $x^2 - 8x - 20$

q. $x^2 - 14x + 49$

r. $x^2 - 6x - 27$

s. $x^2 - 8x + 15$

t. $x^2 + 2x - 24$

u. $x^2 + 9x - 10$

v. $x^2 - 13x + 40$

w. $x^2 + 4x - 21$

x. $x^2 - 8x + 16$

y. $x^2 + 2x - 8$

z. $x^2 - 13x + 30$

23. Factorise the following expressions fully.

a. $2x^2 + 5x + 3$

b. $2x^2 + 7x + 3$

c. $6x^2 + 7x + 2$

d. $3x^2 + 14x + 15$

e. $12x^2 - 8x + 1$

f. $2x^2 - 7x + 3$

g. $8x^2 - 14x + 5$

h. $3x^2 - 2x - 8$

i. $3x^2 - 5x - 2$

j. $5x^2 + 4x - 1$

k. $2x^2 + x - 1$

l. $3x^2 - 2x - 1$

m. $8x^2 + 2x - 3$

n. $8x^2 + 10x - 3$

o. $12x^2 - 11x - 5$

p. $4x^2 + 12x + 9$

q. $24x^2 + 2x - 1$

r. $1 + 3x - 18x^2$

s. $15 - 7x - 2x^2$

t. $x^2 + 8xy + 12y^2$

u. $x^2 - 10xy + 24y^2$

v. $x^2 + 3xc + 2c^2$

w. $x^2 - 5ax - 14a^2$

x. $2x^2 - 5xy - 3y^2$

y. $9x^2 + 6xh - 8h^2$

z. $9x^2 - 12ax + 4a^2$

24. Factorise the following expressions fully.

a. $ab + ac + ad$

b. $3p^2 - 12$

c. $2t^2 - 18$

d. $2x^2 - 32x$

e. $50 - 2x^2$

f. $2p^2 + 3p - 5$

g. $14x^2 - 56n^2$

h. $16x^2 - 8x + 1$

i. $3p^2q - 9pq^2$

j. $1 - 2u + u^2$

k. $3x^3 - 27x$

l. $6a^2 + 5a - 6$

m. $4x^2 + 4x - 8$

n. $10w - 40w^2$

o. $17 - 51x$

p. $4x^2 - 24x + 36$

q. $2x^2 - 12x + 36$

r. $p^7 - p^6 - p^5$

s. $x^4 - 81$

t. $9xy - 45x + 3y$

u. $3x^2 - 147$

v. $12x^2 + 14x - 6$

w. $3x^2 - 24x - 27$

x. $16x^2 - 16x + 4$

y. $24xy + 18y$

z. $a^2 + 4ab - 5b^2$

25. Express each of the following in the form $(x + p)^2 + q$.

a. $x^2 - 2x + 4$

b. $x^2 + 4x + 5$

c. $x^2 - 6x + 11$

d. $x^2 + 2x + 5$

e. $x^2 - 4x + 8$

f. $x^2 + 6x + 10$

g. $x^2 + 2x + 3$

h. $x^2 - 2x + 6$

i. $x^2 + 8x - 3$

j. $x^2 + 14x + 1$

k. $x^2 - 2x - 5$

l. $x^2 + 7x - 1$

m. $x^2 + 13x + 3$

n. $x^2 - x - x$

o. $x^2 + 8x + 13$

p. $x^2 - 4x + 7$

26. Express each of the following in the form $-(x+p)^2 + q$.

    a. $3 - 6x - x^2$ **b.** $1 - 2x - x^2$

    c. $7 + 4x - x^2$ **d.** $5 + 8x - x^2$

    e. $4 - 3x - x^2$ **f.** $2 + x - x^2$

    g. $-x^2 + 6x - 10$ **h.** $-x^2 - 2x - 3$

    i. $-x^2 + 4x - 1$ **j.** $-x^2 + 8x - 17$

    k. $-x^2 + 2x - 3$ **l.** $-x^2 + 8x - 3$

27. Using the answers from **Q25 and Q26,** identify the turning point and state whether it is a maximum or minimum.

ALGEBRAIC FRACTIONS

1 Use mathematical operational skills linked to expressions and formulae by:
EF1.3 Applying algebraic skills to algebraic fractions

28. Express the following algebraic fractions in their simplest form:

    a. $\dfrac{x^3}{x}$ **b.** $\dfrac{(x+1)(x+2)}{x+1}$

    c. $\dfrac{(x+3)(x-3)}{(x-3)(x+1)}$ **d.** $\dfrac{x+4}{(x+4)(x-3)}$

    e. $\dfrac{x^2(x-5)}{x(x-5)^2}$ **f.** $\dfrac{(x+4)(x-3)}{x^2(4+x)}$

    g. $\dfrac{8(x-2)^2}{2(x-9)(x-2)}$ **h.** $\dfrac{8xyz^2}{6xz}$

    i. $\dfrac{21(x^2-2x+1)}{14x(x^2-2x+1)}$ **j.** $\dfrac{(2x+1)^2}{(2x+1)(x-5)}$

    k. $\dfrac{(3x+7)^2}{(3x+6)(3x+7)}$ **l.** $\dfrac{(1-2x)(1+3x)^3}{(1+3x)^2\,(1-2x)^2}$

29. By first factorising, express the following algebraic fractions in their simplest form:

a. $\dfrac{3x-6}{9}$

b. $\dfrac{4}{4b-20}$

c. $\dfrac{2x-10}{2x+10}$

d. $\dfrac{x+2}{4x+8}$

e. $\dfrac{x-2}{3x-6}$

f. $\dfrac{3-3x}{7-7x}$

g. $\dfrac{a^2-ab}{5a-5b}$

h. $\dfrac{x-3}{x^3-3x^2}$

i. $\dfrac{x-3}{x^2-9}$

j. $\dfrac{3x+15}{x^2-25}$

k. $\dfrac{9x^2+9x}{9x}$

l. $\dfrac{x-2}{x^2-3x+2}$

m. $\dfrac{x^2+6x+9}{x+3}$

n. $\dfrac{x^2+5x}{x^2+4x-5}$

o. $\dfrac{4x^2-9x+2}{x-2}$

p. $\dfrac{x^2-1}{x^2+8x+7}$

q. $\dfrac{3x^2-3y^2}{x^2-2xy+y^2}$

r. $\dfrac{x^2+2x-15}{5x+25}$

s. $\dfrac{3x^2+11x-4}{2x+8}$

t. $\dfrac{10x-5}{2x^2-3x+1}$

u. $\dfrac{x^2-5x+6}{x^2-2x-3}$

v. $\dfrac{4x^2+2x-6}{2x^2-4x+2}$

w. $\dfrac{px+py}{x^2-y^2}$

x. $\dfrac{m-m^2}{m^3-m^2}$

y. $\dfrac{p^2-4pq+4q^2}{(p-2q)^2}$

z. $\dfrac{x^4-1}{x^2-1}$

30. Simplify these expressions to a single fraction

a. $\dfrac{3}{2x} - \dfrac{2}{5y}$

b. $\dfrac{4}{5z} + \dfrac{2}{3y}$

c. $\dfrac{3x}{z} + \dfrac{5x}{y}$

d. $\dfrac{m}{3a} - \dfrac{q}{4a}$

e. $\dfrac{2}{g^2} + \dfrac{4}{g}$

f. $\dfrac{2}{t^2} - \dfrac{1}{t}$

g. $\dfrac{7}{x} - \dfrac{2}{x^2}$

h. $\dfrac{8}{h} - \dfrac{16}{h^3}$

i. $\dfrac{4}{x^2} - \dfrac{3}{5x}$

j. $\dfrac{9}{5t} - \dfrac{7}{4t^2}$

k. $\dfrac{3x}{5y} - \dfrac{2y}{7x}$

l. $\dfrac{m}{7n} + \dfrac{3n}{5m}$

31. Simplify these expressions to a single fraction

a. $\dfrac{2}{x-3} + \dfrac{3}{x+2}$

b. $\dfrac{4}{x-2} + \dfrac{3}{x+5}$

c. $\dfrac{6}{x-2} - \dfrac{2}{x+1}$

d. $\dfrac{5}{a+2} - \dfrac{3}{a-1}$

e. $\dfrac{1}{g-7} - \dfrac{6}{g-2}$

f. $\dfrac{2}{t} + \dfrac{1}{t-1}$

g. $\dfrac{7}{x} - \dfrac{1}{x-3}$

h. $\dfrac{8}{h-2} + \dfrac{3}{h}$

i. $\dfrac{8}{x} - \dfrac{2}{x^2}$

j. $\dfrac{t-3}{t^2} + \dfrac{7}{2t}$

k. $\dfrac{4}{5y} - \dfrac{15(y-2)}{2y^2}$

l. $\dfrac{9}{n} - \dfrac{2-n}{3n^2}$

m. $\dfrac{p}{p-6} + \dfrac{p}{6-p}$

n. $\dfrac{m}{3-m} - \dfrac{m}{m-3}$

32. Simplify these expressions to a single fraction

a. $\dfrac{p}{4} \times \dfrac{p}{2}$

b. $\dfrac{5}{a} \times \dfrac{2}{a}$

c. $\dfrac{x}{6} \times \dfrac{2}{x}$

d. $\dfrac{3c}{7} \times \dfrac{14}{c}$

e. $\dfrac{5g}{3} \times \dfrac{4g}{10}$

f. $\dfrac{1}{x} \div \dfrac{3}{x}$

g. $\dfrac{3m}{7} \div \dfrac{15m}{56}$

h. $\dfrac{7b}{3} \div \dfrac{7b}{9}$

i. $\dfrac{4}{w} \div \dfrac{2}{9w}$

j. $\dfrac{8k}{7} \div \dfrac{2}{3k}$

k. $\dfrac{11}{9d} \div \dfrac{11d}{18}$

l. $\dfrac{6g^2}{5} \times \dfrac{15}{2g}$

m. $\dfrac{2q^3}{r} \times \dfrac{5r}{2q^4}$

n. $\dfrac{m^3}{6} \div \dfrac{m^2}{3}$

o. $\dfrac{p}{2q} \div \dfrac{p}{8q}$

p. $\dfrac{k^2}{n} \div \dfrac{k}{n^2}$

q. $\dfrac{1}{b^2} \div \dfrac{5}{b}$

r. $\dfrac{h^2}{t} \div \dfrac{9h}{3t}$

s. $\dfrac{7y}{24x^2} \times \dfrac{6x}{49y}$

t. $\dfrac{2p^2}{5} \div \dfrac{p^3}{10}$

u. $\dfrac{x^2+5x+6}{4x^2-1} \div \dfrac{x^2+4x+3}{2x^2+x-1}$

v. $\dfrac{x^2+x-6}{x^2-3x+2} \times \dfrac{x^2-1}{x^2+5x+4}$

w. $\dfrac{x^2+x-12}{x^2+8x+16} \div \dfrac{x^2-4x+3}{x^2+2x-8}$

**x.** $\dfrac{6x^2+7x+2}{3x^2-27x+60} \times \dfrac{6x^2-18x-24}{3x^2-7x-6}$

y. $\dfrac{1-x^2}{2x^2+x-6} \times \dfrac{x^2-x-6}{x^2-2x-3}$

z. $\dfrac{x^3-6x^2+8x}{x^2-9} \div \dfrac{x^2-2x}{x^2+6x+9}$

GRADIENT

1 Use mathematical operational skills linked to expressions and formulae by:
EF1.4 Applying geometric skills linked to the use of formulae

33. Calculate the gradient of the lines below.

a *b* *c* *d*

e *f* *g* *h*

i *j* *k* *l*

m *n* *o* *p*
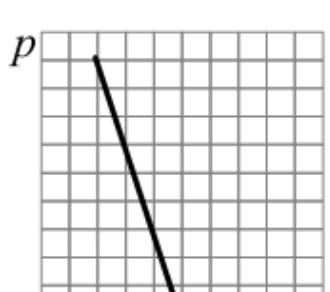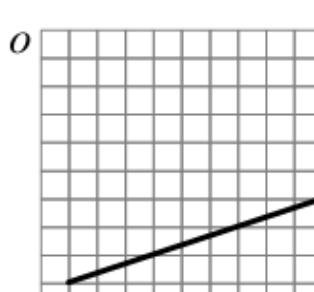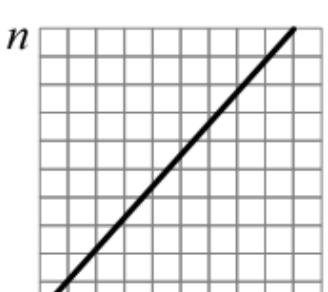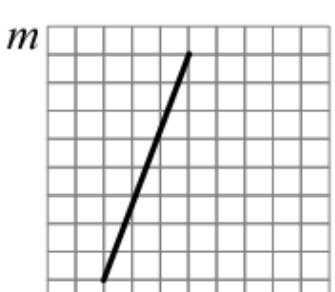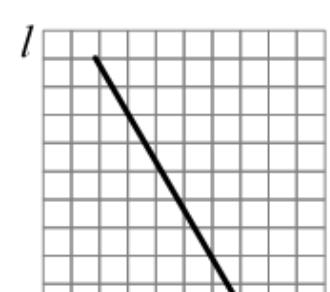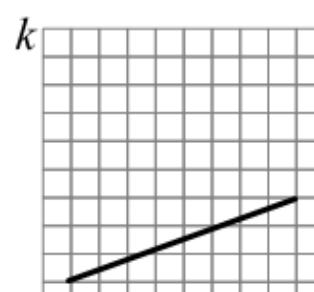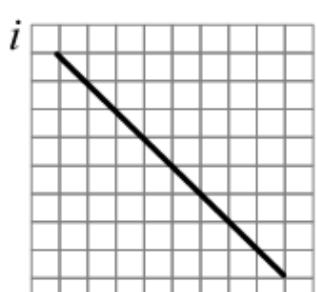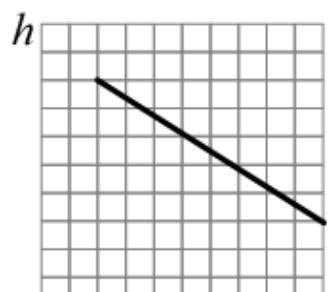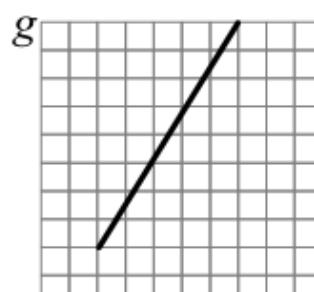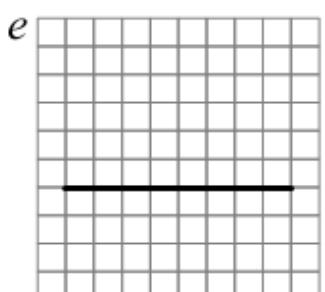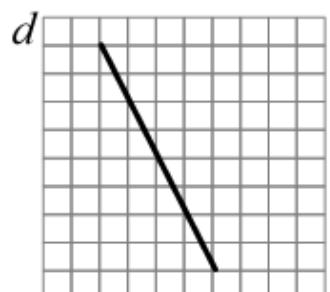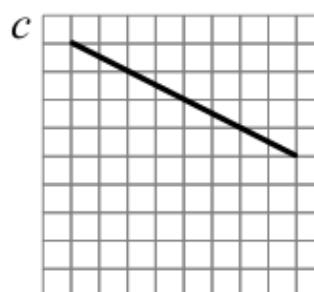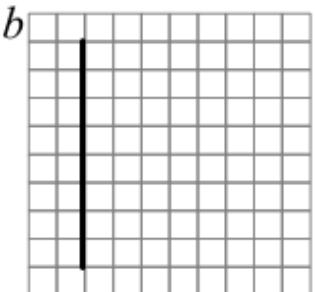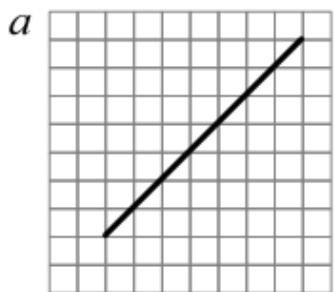
34. Calculate the gradients of the lines below

$B(-5,2)$

$A(5,-6)$

$D(3,7)$

$C(-2,3)$

$E(7,-3)$

$F(2,1)$

$H(4,7)$

$G(2,2)$

$I(-3,2)$

$J(5,-3)$

$K(-4,6)$

$L(-9,-1)$

$N(15,2)$

$M(6,-1)$

$P(3,8)$

$O(5,-3)$

$Q(5,10)$

$R(8,-2)$

$X(0,12)$

$T(9,0)$

$S(-4,-1)$

$V(0,0)$

$U(10,0)$

$W(0,-4)$
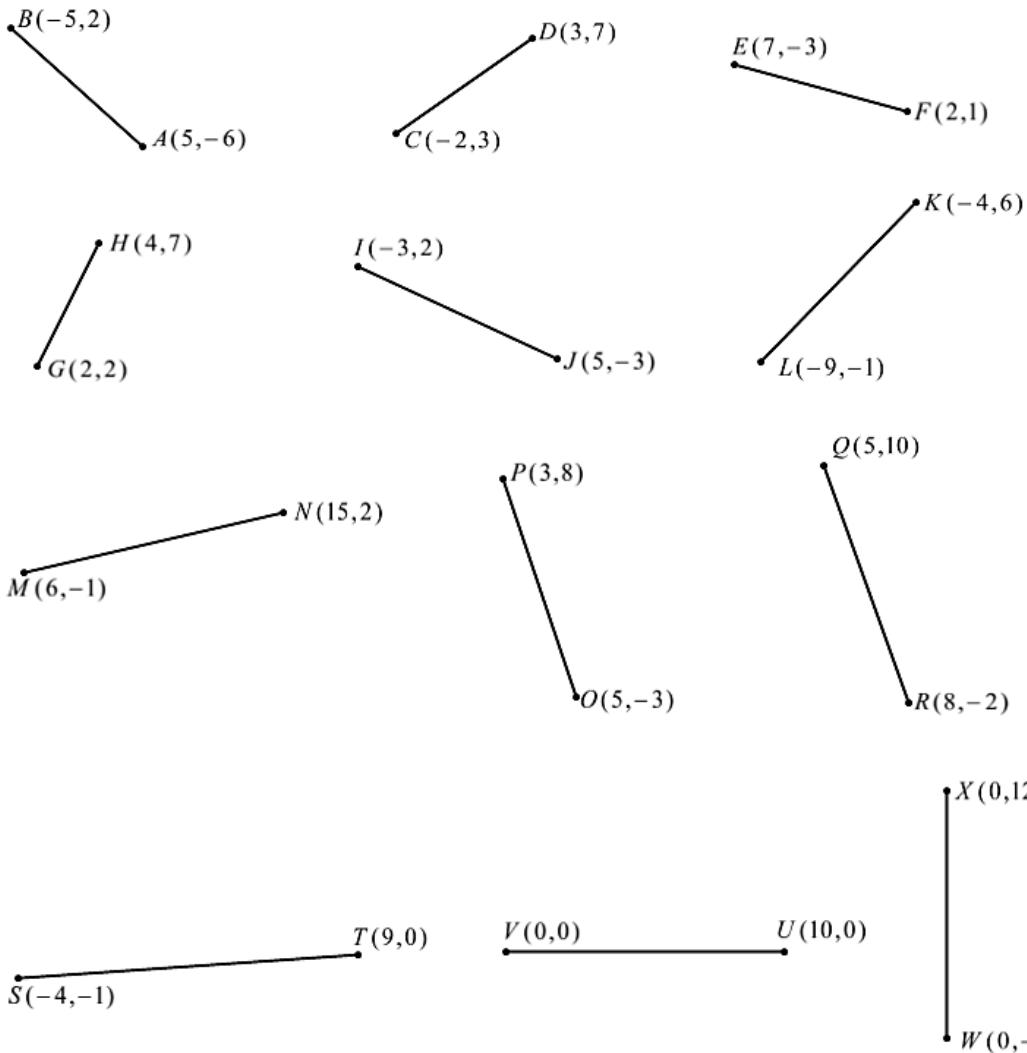
35. Calculate the gradient of the lines which pass through the following pairs of points.

a. $(3,-1)$ and $(6,4)$

b. $(-5,3)$ and $(2,7)$

c. $(8,-4)$ and $(3,2)$

d. $(-6,-3)$ and $(3,6)$

e. $(11,-2)$ and $(6,0)$

f. $(0,-1)$ and $(11,10)$

g. $(1,-7)$ and $(5,2)$

h. $(3,-5)$ and $(2,6)$

i. $(4,-1)$ and $(3,6)$

j. $(2,4)$ and $(2,11)$

k. $(6,2)$ and $(11,4)$

l. $(8,1)$ and $(-8,2)$

m. $(7,4)$ and $(15,4)$

n. $(-3,1)$ and $(7,-3)$

o. $(5,2)$ and $(-3,4)$

p. $(2,4)$ and $(-7,1)$

ARC LENGTH AND SECTOR AREA

1 Use mathematical operational skills linked to expressions and formulae by:
EF1.4 Applying geometric skills linked to the use of formulae

36. For each of the following calculate:
  i.   The arc length
         1.   As an exact value in Pi
         2.   To 3 significant figures
  ii. The sector area
         1.   As an exact value in Pi
         2.   To 3 significant figures
  iii. The perimeter of the shape to 3 significant figures.

a

8 cm

294°

b

98°

8 cm

c

3.6 cm

237°

d

46°

1.6 m

e

14.3 cm

63°

f

3.1 mm
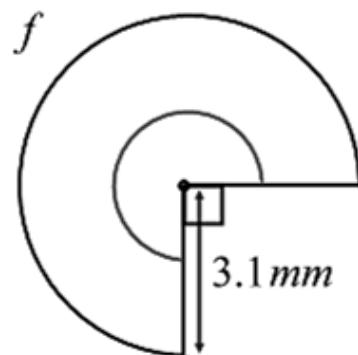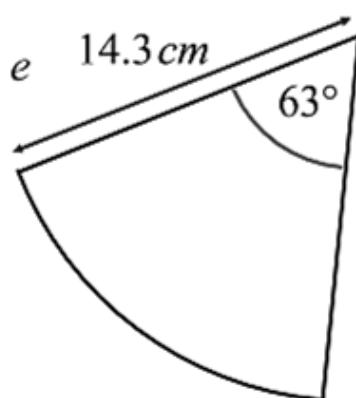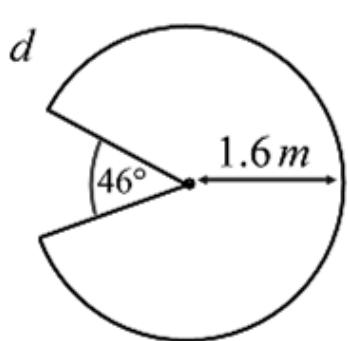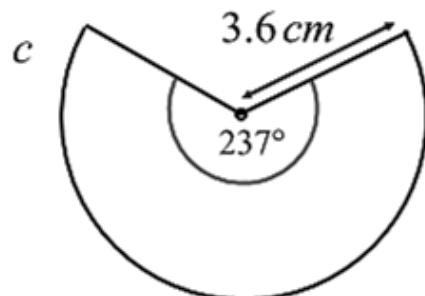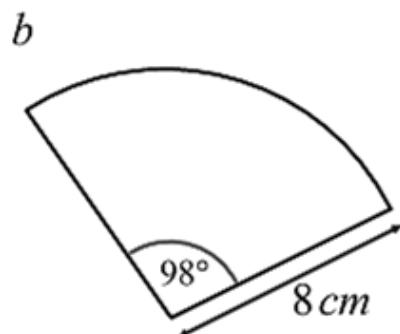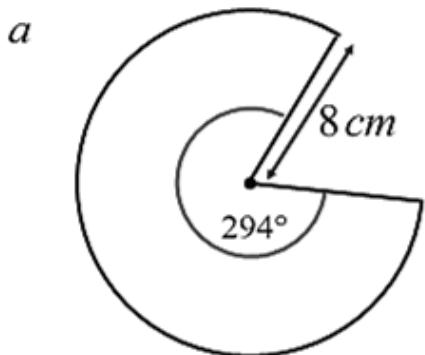
VOLUMES OF SOLIDS

1 Use mathematical operational skills linked to expressions and formulae by:
EF1.4 Applying geometric skills linked to the use of formulae

**37.** Calculate the volumes of the Hemispheres and Spheres:
  i. As an exact value using Pi
  ii. To 3 significant figures

a

b

c

10 cm

1.2 m

15 cm

d

e

f

3.1 cm

8.25 cm

18 mm
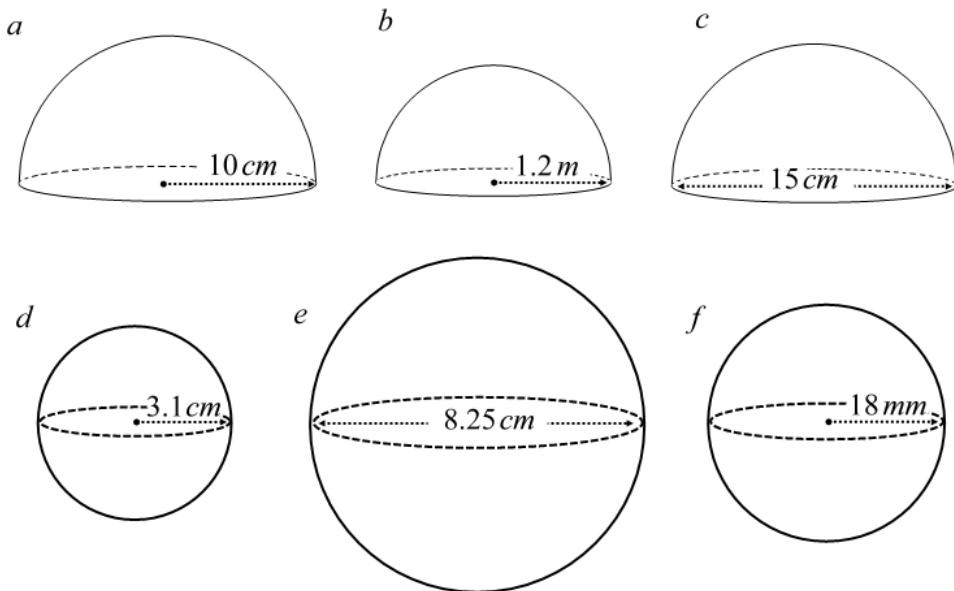
38. Calculate the volumes of the Cylinders:
  i. As an exact value using Pi
  ii. To 3 significant figures

a

4 cm

4 cm

b

1.4 m

c

5 mm

8.3 m

65 mm

d

3.6 m

1.2 m

e

4.2 m

80 cm
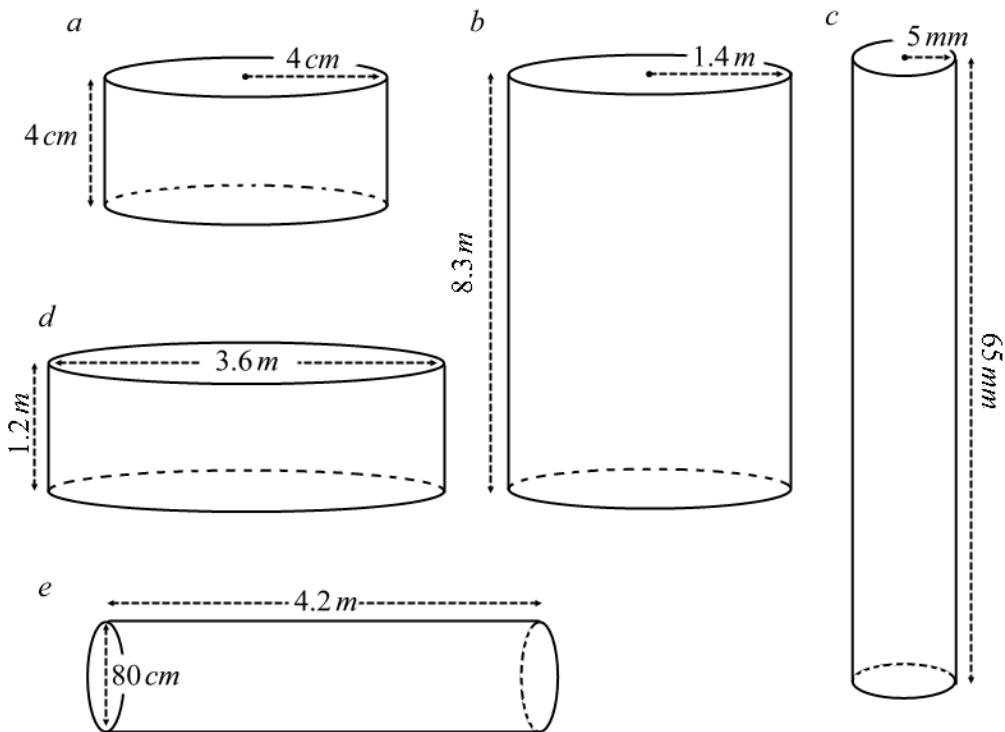
39. Calculate the volumes of the Cones:
   i. As an exact value using Pi
   ii. To 3 significant figures

a
−30 cm−
70 cm

b
54 mm
−18 mm

c
5 cm
··15 cm··

d
11.6 m
− 3.4 m −

e
60 cm
72.5 cm

f
2.4 m
····· 40 cm ·····
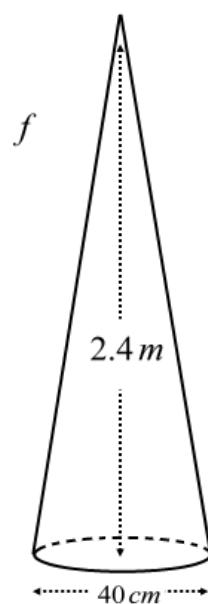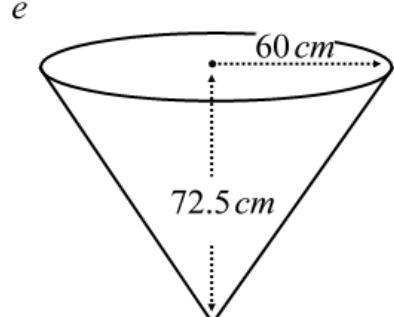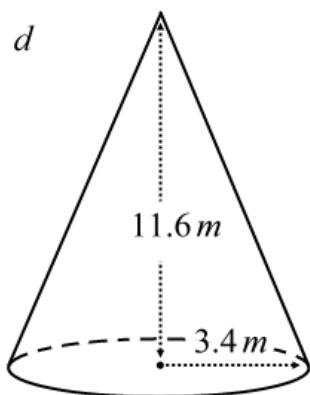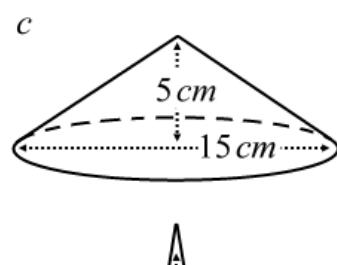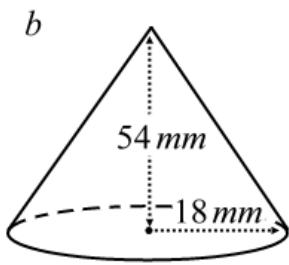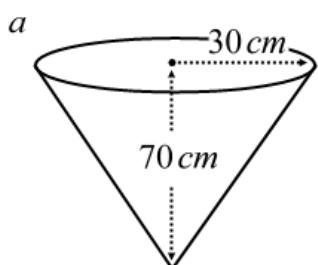
40. Calculate the volumes of these Square Based Pyramids.

a
8 cm
5 cm

b
30 cm
22 cm

c
6.8 cm
4.1 cm

d
35.2 m
20 m

e
68 cm
1.4 m

f
83 mm
47 mm
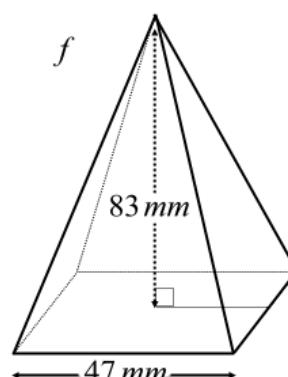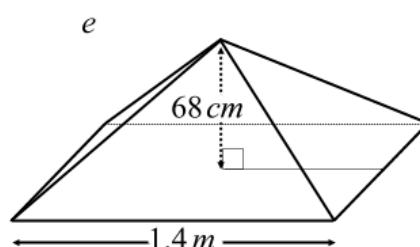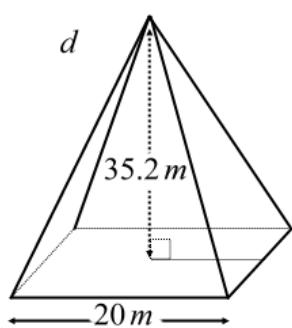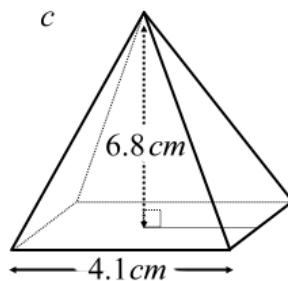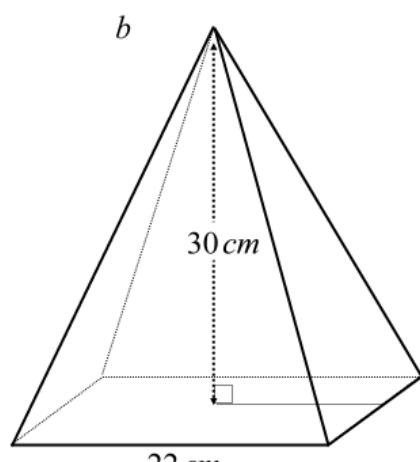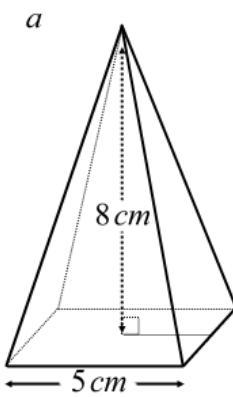
ANSWERS
SURDS

1.

a. $\sqrt{20} = \sqrt{(4 \times 5)} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$
b. $\sqrt{75} = \sqrt{(25 \times 3)} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$
c. $\sqrt{96} = \sqrt{(16 \times 6)} = \sqrt{16} \times \sqrt{6} = 4\sqrt{6}$
d. $\sqrt{128} = \sqrt{(64 \times 2)} = \sqrt{64} \times \sqrt{2} = 8\sqrt{2}$
e. $\sqrt{175} = \sqrt{(25 \times 7)} = \sqrt{25} \times \sqrt{7} = 5\sqrt{7}$
f. $\sqrt{300} = \sqrt{(100 \times 3)} = \sqrt{100} \times \sqrt{3} = 10\sqrt{3}$
g. $\sqrt{45} = \sqrt{(9 \times 5)} = \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$

2.

a)	$3\sqrt{2}$	**b)**	$2\sqrt{2}$	**c)**	$2\sqrt{6}$	**d)**	$3\sqrt{3}$	**e)**	$10\sqrt{2}$
f)	$4\sqrt{2}$	**g)**	$6\sqrt{2}$	**h)**	$7\sqrt{3}$	**i)**	$8\sqrt{2}$	**j)**	$3\sqrt{5}$
k)	$2\sqrt{7}$	**l)**	$9\sqrt{5}$	**m)**	$6\sqrt{5}$	**n)**	$4\sqrt{7}$	**o)**	$7\sqrt{5}$

3.

a)	$8\sqrt{2}$	**b)**	$6\sqrt{11}$
c)	$\sqrt{13}$	**d)**	$3\sqrt{2}$
e)	$\sqrt{5}$	**f)**	$2\sqrt{6}$
g)	$2\sqrt{2}$	**h)**	$5\sqrt{6}$
i)	$6\sqrt{3}$	**j)**	$6\sqrt{5}$
k)	$5\sqrt{11}$	**l)**	$5\sqrt{10}$
m)	$3\sqrt{7} + 11\sqrt{2}$	**n)**	$12\sqrt{3} + 12\sqrt{2}$
o)	$4\sqrt{3} + 17\sqrt{2}$		

4.

a)	3	**b)**	7	**c)**	20	**d)**	110
e)	90	**f)**	4	**g)**	10	**h)**	$4\sqrt{10}$

5.

a) 30   **b)** $48\sqrt{3}$ **c)** $30\sqrt{2}$

d) $24\sqrt{5}$ **e)** 80   **f)** $36\sqrt{2}$

6.

a) 2   **b)** $2\sqrt{2}$ **c)** $2\sqrt{10}$ **d)** 3   **e)** 5

f) $2\sqrt{2}$ **g)** 5   **h)** $2\sqrt{3}$ **i)** $2\sqrt{5}$ **j)** $3\sqrt{2}$

k) $\dfrac{1}{2\sqrt{2}}$ **l)** $\dfrac{5}{3}$ **m)** $\dfrac{1}{2\sqrt{10}}$ **n)** $\dfrac{1}{2\sqrt{5}}$ **o)** $\dfrac{1}{5\sqrt{2}}$

7.

a) $\dfrac{2}{3}$ **b)** $\dfrac{2}{3}$ **c)** $\dfrac{2}{3}$ **d)** $\dfrac{3}{4}$ **e)** $\dfrac{2}{3}$

f) $\dfrac{6}{7}$ **g)** $\dfrac{9}{7}$ **h)** $\dfrac{8}{5}$ **i)** $\dfrac{4}{3}$ **j)** $\dfrac{2}{5}$

k) $\dfrac{2}{5}$ **l)** $\dfrac{3}{2}$ **m)** $\dfrac{2}{3}$ **n)** $\dfrac{5}{7}$ **o)** $\dfrac{1}{2}$

8.

a) $=\sqrt{\dfrac{16}{100}}=\dfrac{2}{5}$ **b)** $=\sqrt{\dfrac{36}{100}}=\dfrac{3}{5}$ **c)** $=\sqrt{\dfrac{64}{100}}=\dfrac{4}{5}$ **d)** $=\sqrt{\dfrac{4}{100}}=\dfrac{1}{5}$

e) $=\sqrt{\dfrac{25}{10000}}=\dfrac{1}{20}$ **f)** $=\sqrt{\dfrac{9}{10000}}=\dfrac{3}{100}$ **g)** $=\sqrt{\dfrac{49}{1000000}}=\dfrac{7}{1000}$ **h)** $=\sqrt{\dfrac{1}{1000000}}=\dfrac{1}{1000}$

INDICES

9.

a) x^3 **b)** t^5 **c)** b^3 **d)** a^4

e) g^{13} **f)** d^4 **g)** v^{16} **h)** $h^0=1$

i) m^6 **j)** d^{15} **k)** z^6 **l)** p^{15}

m) g^{13} **n)** j^6 **o)** h^{10} **p)** $y^{17/2}$

q) 5^{17} **r)** 13^{21} **s)** 8^{19} **t)** 4^{20}

10.

a)	$6x^2$	**b)**	$10t^3$	**c)**	$3b^6$	**d)**	$4a^{10}$
e)	$26g^5$	**f)**	$64d^4$	**g)**	$-12v^7$	**h)**	$-8h^5$
i)	$-2m^3$	**j)**	$8d^{11}$	**k)**	$156z^5$	**l)**	$56p^{15}$
m)	$12g^0 = 12$	**n)**	$36j^{-3}$	**o)**	$-12h^{-8}$	**p)**	$18y^{3/2}$
q)	$60f^5$	**r)**	$20x^3$	**s)**	$-162m^{15}$	**t)**	$128x^7$

11.

a)	x^5	**b)**	x^3	**c)**	x^6	**d)**	x^5
e)	x^{11}	**f)**	a^9	**g)**	b^5	**h)**	z^4
i)	y^3	**j)**	z^3	**k)**	m	**l)**	1
m)	c^4	**n)**	h	**o)**	f^8	**p)**	q^{15}
q)	$3k$	**r)**	$5r^2$	**s)**	$9n^3$	**t)**	$\frac{3}{2}x^{5/2}$
u)	0.6^5	**v)**	1.2^5	**w)**	$\left(\frac{2}{3}\right)^2 = \frac{4}{9}$	**x)**	$(3x)^3$

12.

a)	x^3	**b)**	t^7	**c)**	b	**d)**	$2a^5$
e)	$\dfrac{9g^4}{2}$	**f)**	$\dfrac{51d^3}{4}$	**g)**	$\dfrac{9v}{2}$	**h)**	$\dfrac{8h^4}{3}$
i)	$\dfrac{12m^2}{7}$	**j)**	$5d^6$	**k)**	$\dfrac{-3z^2}{2}$	**l)**	$\dfrac{p^7}{3}$

13.

a $\quad a^{\frac{1}{2}}$

b $\quad b^{\frac{11}{3}}$

c $\quad \dfrac{1}{c} \ or \ c^{-1}$

d $\quad d^{\frac{20}{3}}$

e $\quad \dfrac{1}{E} \ or \ E^{-1}$

f $\quad f$

g $\quad g^{\frac{3}{2}}$

h $\quad \dfrac{1}{h^{1/12}} \ or \ h^{-1/12}$

SCIENTIFIC NOTATION

14.

a)	3×10^{11}	**b)**	8×10^{7}	**c)**	7×10^{8}	**d)**	2×10^{9}
e)	4.2×10^{7}	**f)**	2.1×10^{10}	**g)**	3.7×10^{9}	**h)**	6.3×10^{2}

15.

a)	1.25×10^{-1}	**b)**	2.4×10^{-3}	**c)**	1.2×10^{-7}	**d)**	2.49×10^{-6}
e)	4.12×10^{-3}	**f)**	4.9×10^{-4}	**g)**	3.07×10^{-7}	**h)**	2.0107×10^{-3}

16.

a)	2300	**b)**	1040000	**c)**	156000	**d)**	90300000
e)	4100000000	**f)**	600.01	**g)**	3560000	**h)**	712000000000

17.

   a)   0.034   **b)**   0.000006021   **c)**   0.00008201   **d)**   0.006549

   e)   0.00000013   **f)**   0.000000005001   **g)**   0.00799   **h)**   0.409

18.

a	$1874 \; or \; 1.874 \times 10^3$		**b**	$36830000 \; or \; 3.683 \times 10^7$
c	$718400 \; or \; 7.184 \times 10^5$		**d**	$145000 \; or \; 1.45 \times 10^5$
e	$5658000 \; or \; 5.658 \times 10^6$		**f**	$85660 \; or \; 8.566 \times 10^4$
g	$2210000 \; or \; 2.21 \times 10^6$		**h**	$48900000 \; or \; 4.89 \times 10^7$
i	$999900000 \; or \; 9.999 \times 10^8$		**j**	$99 \; or \; 9.9 \times 10$
k	$10200000 \; or \; 1.02 \times 10^7$		**l**	$374000000000 \; or \; 3.74 \times 10^{11}$
m	7.8		**n**	$300000000 \; or \; 3.0 \times 10^8$
o	$40 \; or \; 4.0 \times 10$		**p**	$7000000 \; or \; 7.0 \times 10^6$

19.

    a. 333165.8291 …

    b. 77.63557858 …

    c. 1047.945205 …

    d. 9.299065421 …

    e. 9.582219251 …

f.	Mercury	$0.0536 \ldots hours$	$\approx 3.2 \; minutes$
	Venus	$0.1002 \ldots hours$	$\approx 6 \; minutes$
	Earth	$0.1386 \ldots hours$	$\approx 8.3 \; minutes$
	Mars	$0.1914 \ldots hours$	$\approx 11.5 \; minutes$
	Jupiter	$0.7214 \ldots hours$	$\approx 43.3 \; minutes$
	Saturn	$1.3282 \ldots hours$	$\approx 1 \; hour \; and \; 19.7 \; minutes$
	Uranus	$2.6615 \ldots hours$	$\approx 2 \; hour \; and \; 39.7 \; minutes$
	Neptune	$4.1650 \ldots hours$	$\approx 4 \; hour \; and \; 9.9 \; minutes$
	Pluto	$5.4926 \ldots hours$	$\approx 5 \; hour \; and \; 29.6 \; minutes$
g.	Difference	$2.7232 \ldots hours$	$\approx 2 \; hour \; and \; 43.4 \; minutes$

BRACKETS AND EQUATIONS

20.

   a) $7x^2 - 14xy$ **b)** $4x^3 + 8x^2 - 28x$ **c)** $ax^3 - 3ax^2$ **d)** $4x - 12$

   e) $x^2 + 8x + 16$ **f)** $4x^2 - 12x + 9$ **g)** $x^2 - 8x + 16$ **h)** $x^4 + 2x^2 + 1$

   i) $x^2 - 2x - 3$ **j)** $2x^2 - 11x - 21$ **k)** $3x^3 - 4x^2 + 3x - 4$ **l)** $-x^2 - x + 12$

   m) $x^2 - 9$ **n)** $4x^2 - 25$ **o)** $49 - x^2$ **p)** $3x^2 - 3$

   q) $x^3 + 2x^2 + 3x + 2$ **r)** $x^3 - 7x + 6$ **s)** $x^3 + 5x^2 + 8x + 4$ **t)** $x + 11$

   u) $13 - 2x$ **v)** $2 - 2x - x^2$ **w)** $x^2 - x - 5$ **x)** $4x^2 + 5x + 6$

21.

   a) $(x + 2)(x - 2)$ **b)** $(x + 4)(x - 4)$ **c)** $(x + 3)(x - 3)$ **d)** $(x + 3)(x - 3)$

   e) $(2 + x)(2 - x)$ **f)** $(3 + x)(3 - x)$ **g)** $(7 + x)(7 - x)$ **h)** $3(x + 1)(x - 1)$

   i) $2(x + 3)(x - 3)$ **j)** $(2x + 5)(2x - 5)$ **k)** $(5x + 1)(5x - 1)$ **l)** $(1 + 4x)(1 - 4x)$

   m) $7(x + 2)(x - 2)$ **n)** $(3x + 11)(3x - 11)$ **o)** $\dfrac{1}{2}(x + 2)(x - 2)$ **p)** $\dfrac{1}{4}(x + 2)(x - 2)$

22.

a. $(x-1)^2$

b. $(z-3)^2$

c. $(x+5)(x+2)$

d. $(x-3)(x-6)$

e. $(x-3)(x-8)$

f. $(x+4)(x-1)$

g. $(x+3)(x-2)$

h. $(x-3)(x+2)$

i. $(x-6)(x+1)$

j. $(x-6)(x+2)$

k. $(x-5)(x+3)$

l. $(x-11)(x+1)$

m. $(x+6)(x-5)$

n. $(x+7)(x-4)$

o. $(x-2)(x-8)$

p. $(x-10)(x+2)$

q. $(x-7)^2$

r. $(x-9)(x+3)$

s. $(x-3)(x-5)$

t. $(x+6)(x-4)$

u. $(x+10)(x-1)$

v. $(x-5)(x-8)$

w. $(x+7)(x-3)$

x. $(x-4)^2$

y. $(x+4)(x-2)$

z. $(x-3)(x-10)$

23.

a. $(2x + 3)(x + 1)$

b. $(2x + 1)(x + 3)$

c. $(3x + 2)(2x + 1)$

d. $(3x + 5)(x + 3)$

e. $(2x - 1)(6x - 1)$

f. $(2x - 1)(x - 3)$

g. $(2x - 1)(4x - 5)$

h. $(3x + 4)(x - 2)$

i. $(3x + 1)(x - 2)$

j. $(5x - 1)(x + 1)$

k. $(2x - 1)(x + 1)$

l. $(3x + 1)(x - 1)$

m. $(4x + 3)(2x - 1)$

n. $(4x - 1)(2x + 3)$

o. $(3x + 1)(4x - 5)$

p. $(2x + 3)^2$

q. $(6x - 1)(4x + 1)$

r. $-(3x - 1)(6x + 1)$

s. $-(2x - 3)(x + 5)$

t. $(x + 2y)(x + 6y)$

u. $(x - 4y)(x - 6y)$

v. $(x + c)(x + 2c)$

w. $(x - 7a)(x + 2a)$

x. $(2x + y)(x - 3y)$

y. $(3x - 2h)(3x + 4h)$

z. $(3x - 2a)^2$

24.

    a. $a(b + c + d)$

    b. $3(p + 2)(p - 2)$

    c. $2(t + 3)(t - 3)$

    d. $2x(x - 16)$

    e. $2(5 - x)(5 + x)$

    f. $(2p + 5)(p - 1)$

    g. $14(x - 2n)(x + 2n)$

    h. $(4x - 1)^2$

    i. $3pq(p - 3q)$

    j. $(u - 1)^2$

    k. $3x(x + 3)(x - 3)$

    l. $(3a - 2)(2a + 3)$

    m. $4(x + 2)(x - 1)$

    n. $10w(1 - 4w)$

    o. $17(1 - 3x)$

    p. $4(x - 3)^2$

    q. $2(x^2 - 6x + 18)$

    r. $p^5(p^2 - p - 1)$

    s. $(x^2 + 9)(x + 3)(x - 3)$

    t. $3(3xy - 15x + y)$

    u. $3(x + 7)(x - 7)$

    v. $2(3x - 1)(2x + 3)$

    w. $3(x - 9)(x + 1)$

    x. $4(2x - 1)^2$

    y. $6y(4x + 3)$

    z. $(a - 5b)(a - b)$

25.

    a. $(x - 1)^2 + 3$

    b. $(x + 2)^2 + 1$

    c. $(x - 3)^2 + 2$

    d. $(x + 1)^2 + 4$

    e. $(x - 2)^2 + 4$

    f. $(x + 3)^2 + 1$

    g. $(x + 1)^2 + 2$

    h. $(x - 1)^2 + 5$

    i. $(x + 4)^2 - 19$

    j. $(x + 7)^2 - 48$

    l. $\left(x + \frac{7}{2}\right)^2 - \frac{53}{4}$

    k. $(x - 1)^2 - 6$

    m. $\left(x + \frac{13}{2}\right)^2 - \frac{157}{4}$

    n. $(x - 1)^2 - 1$

    o. $(x + 4)^2 - 3$

    p. $(x - 2)^2 + 3$

26.

a. $-(x+3)^2 + 12$

b. $-(x+1)^2 + 2$

c. $-(x-2)^2 + 11$

d. $-(x-4)^2 + 21$

e. $-\left(x - \frac{3}{2}\right)^2 + \frac{25}{4}$

f. $-\left(x - \frac{1}{2}\right)^2 + \frac{9}{4}$

g. $-(x-3)^2 - 1$

h. $-(x+1)^2 - 2$

i. $-(x-2)^2 + 3$

j. $-(x-4)^2 - 1$

k. $-(x-1)^2 - 2$

l. $-(x-4)^2 + 13$

27. Using the answers from **Q25** and **Q26,** identify the turning point and state whether it is a maximum or minimum.

	Q 25			Q 26	
a	min	$(1,3)$	**a**	max	$(-3,12)$
b	min	$(-2,1)$	**b**	max	$(-1,2)$
c	min	$(3,2)$	**c**	max	$(2,11)$
d	min	$(-1,4)$	**d**	max	$(4,21)$
e	min	$(2,4)$	**e**	max	$\left(^3/_2, ^{25}/_4\right)$
f	min	$(-3,1)$	**f**	max	$\left(^1/_2, ^9/_4\right)$
g	min	$(-1,2)$	**g**	max	$(3,-1)$
h	min	$(1,5)$	**h**	max	$(-1,-2)$
i	min	$(-4,-19)$	**i**	max	$(2,3)$
j	min	$(-7,-48)$	**j**	max	$(4,-1)$
k	min	$(1,-6)$	**k**	max	$(1,-2)$
l	min	$\left(-^7/_2, -^{53}/_4\right)$	**l**	max	$(4,13)$
m	min	$\left(-^{13}/_2, -^{157}/_4\right)$			
n	min	$(1,-1)$			
o	min	$(-4,-3)$			
p	min	$(2,3)$			

ALGEBRAIC FRACTIONS

28.

a. x^2

b. $x + 2$

c. $\dfrac{(x+3)}{(x+1)}$

d. $\dfrac{1}{x-3}$

e. $\dfrac{x}{x-5}$

f. $\dfrac{x-3}{x^2}$

g. $\dfrac{4(x-2)}{x-9}$

h. $\dfrac{4yz}{3}$

i. $\dfrac{3}{2x}$

j. $\dfrac{2x+1}{x-5}$

k. $\dfrac{3x+7}{3x+6}$

l. $\dfrac{1+3x}{1-2x}$

29.

a. $\dfrac{3(x-2)}{9} = \dfrac{x-2}{3}$

b. $\dfrac{4}{4(b-5)} = \dfrac{1}{b-5}$

c. $\dfrac{2(x-5)}{2(x+5)} = \dfrac{x-5}{x+5}$

d. $\dfrac{x+2}{4(x+2)} = \dfrac{1}{4}$

e. $\dfrac{x-2}{3(x-2)} = \dfrac{1}{3}$

f. $\dfrac{3(1-x)}{7(1-x)} = \dfrac{3}{7}$

g. $\dfrac{a(a-b)}{5(a-b)} = \dfrac{a}{5}$

h. $\dfrac{x-3}{x^2(x-3)} = \dfrac{1}{x^2}$

i. $\dfrac{x-3}{(x+3)(x-3)} = \dfrac{1}{x+3}$

j. $\dfrac{3(x+5)}{(x+5)(x-5)} = \dfrac{3}{x-5}$

k. $\dfrac{9x(x+1)}{9x} = x + 1$

l. $\dfrac{x-2}{(x-1)(x-2)} = \dfrac{1}{x-1}$

m. $\dfrac{(x+3)(x+3)}{x+3} = x + 3$

n. $\dfrac{x(x+5)}{(x+5)(x-1)} = \dfrac{x}{x-1}$

o. $\dfrac{(4x-1)(x-2)}{x-2} = 4x - 1$

p. $\dfrac{(x+1)(x-1)}{(x+1)(x+7)} = \dfrac{x-1}{x+7}$

q. $\dfrac{3(x+y)(x-y)}{(x-y)(x-y)} = \dfrac{3(x+y)}{x-y}$

r. $\dfrac{(x+5)(x-3)}{5(x+5)} = \dfrac{x-3}{5}$

s. $\dfrac{(3x-1)(x+4)}{2(x+4)} = \dfrac{3x-1}{2}$

t. $\dfrac{5(2x-1)}{(2x-1)(x+1)} = \dfrac{5}{x+1}$

u. $\dfrac{(x-2)(x-3)}{(x-3)(x+1)} = \dfrac{x-2}{x+1}$

v. $\dfrac{2(2x+3)(x-1)}{2(x-1)(x-1)} = \dfrac{2x+3}{x-1}$

w. $\dfrac{p(x+y)}{(x+y)(x-y)} = \dfrac{p}{x-y}$

x. $\dfrac{m(1-m)}{m^2(m-1)} = -\dfrac{1}{m}$

y. $\dfrac{(p-2q)(p-2q)}{(p-2q)^2} = 1$

z. $\dfrac{(x^2+1)(x^2-1)}{x^2-1} = x^2 + 1$

30.

a. $\dfrac{15y-4x}{10xy}$

b. $\dfrac{12y+10z}{15yz}$

c. $\dfrac{3xy+5xz}{yz}$

d. $\dfrac{4m-3q}{12a}$

e. $\dfrac{2+4g}{g^2}$

f. $\dfrac{2-t}{t^2}$

g. $\dfrac{7x-2}{x^2}$

h. $\dfrac{8h^2-16}{h^3}$

i. $\dfrac{20-3x}{5x^2}$

j. $\dfrac{36t-35}{20t^2}$

k. $\dfrac{21x^2-10y^2}{35xy}$

l. $\dfrac{5m^2+21n^2}{35mn}$

31.

a. $\dfrac{5x-7}{(x-3)(x+2)}$

b. $\dfrac{7x+14}{(x-2)(x+5)}$

c. $\dfrac{4x+10}{(x-2)(x+1)}$

d. $\dfrac{2a-11}{(a+2)(a-1)}$

e. $\dfrac{40-5g}{(g-7)(g-2)}$

f. $\dfrac{3t-2}{t(t-1)}$

g. $\dfrac{6x-21}{x(x-3)}$

h. $\dfrac{11h-6}{h(h-2)}$

i. $\dfrac{8x-2}{x^2}$

j. $\dfrac{3t-6}{2t^2}$

k. $\dfrac{150+8y-75y^2}{10y^2}$

l. $\dfrac{28n-2}{3n^2}$

m. 0

n. $\dfrac{2m}{3-m}$

32.

a. $\dfrac{p^2}{8}$

b. $\dfrac{10}{a^2}$

c. $\dfrac{1}{3}$

d. 6

e. $\dfrac{2g^2}{3}$

f. $\dfrac{1}{3}$

g. $\dfrac{24}{5}$

h. 3

i. $\dfrac{9}{2}$

j. $\dfrac{12k^2}{7}$

k. $\dfrac{2}{d^2}$

l. $9g$

m. $\dfrac{5}{q}$

n. $\dfrac{m}{2}$

o. 4

p. kn

q. $\dfrac{1}{5b}$

r. $\dfrac{h}{3}$

s. $\dfrac{1}{28x}$

t. $\dfrac{4}{p}$

u. $\dfrac{x+2}{2x+1}$ factorise first

v. $\dfrac{x+3}{x+4}$

w. $\dfrac{x-2}{x-1}$

x. $\dfrac{2(2x+1)(x+1)}{(x-5)(x-3)}$

y. $\dfrac{1-x}{2x-3}$

z. $\dfrac{(x+3)(x-4)}{x-3}$

GRADIENT

33.

a	1	**b**	*undefined*	**c**	$-\dfrac{1}{2}$	**d**	-2
e	0	**f**	-7	**g**	$\dfrac{8}{5}$	**h**	$-\dfrac{5}{8}$
i	-1	**j**	$-\dfrac{1}{8}$	**k**	$\dfrac{3}{8}$	**l**	$-\dfrac{9}{5}$
m	$\dfrac{8}{3}$	**n**	$\dfrac{9}{10}$	**o**	$\dfrac{1}{3}$	**p**	-3

34.

a	$-\dfrac{5}{4}$	**b**	$\dfrac{4}{5}$	**c**	$-\dfrac{4}{5}$
d	$\dfrac{5}{2}$	**e**	$-\dfrac{5}{8}$	**f**	$\dfrac{7}{5}$
g	$\dfrac{1}{3}$	**h**	$-\dfrac{11}{2}$	**i**	-4
j	$\dfrac{1}{13}$	**k**	0	**l**	*undefined*

35.

a	$\dfrac{5}{3}$	**b**	$\dfrac{4}{7}$	**c**	$-\dfrac{6}{5}$	**d**	1
e	$-\dfrac{2}{5}$	**f**	$\dfrac{11}{10}$	**g**	$\dfrac{9}{4}$	**h**	-11
i	-7	**j**	*undefined*	**k**	$\dfrac{2}{5}$	**l**	$-\dfrac{1}{16}$
m	0	**n**	$-\dfrac{2}{5}$	**o**	$-\dfrac{1}{4}$	**p**	$\dfrac{1}{3}$

ARC LENGTH AND SECTOR AREA

36.

	Arc Length		Sector Area		Perimeter
	in terms of π	3 sig fig	in terms of π	3 sig fig	3 sig fig
a	$\dfrac{196}{15}\pi\ cm$	41.1 cm	$\dfrac{784}{15}\pi\ cm^2$	164 cm^2	57.1 cm
b	$\dfrac{196}{45}\pi\ cm$	13.7 cm	$\dfrac{784}{45}\pi\ cm^2$	54.7 cm^2	29.7 cm
c	$\dfrac{237}{50}\pi\ cm$	14.9 cm	$\dfrac{2133}{250}\pi\ cm^2$	26.8 cm^2	22.1 cm
d	$\dfrac{628}{225}\pi\ m$	8.77 m	$\dfrac{2512}{1125}\pi\ m^2$	7.01 m^2	12.0 m
e	$\dfrac{1001}{200}\pi\ cm$	15.7 cm	$\dfrac{1288287}{36000}\pi\ cm^2$	112 cm^2	44.3 cm
f	$\dfrac{93}{20}\pi\ mm$	14.6 mm	$\dfrac{2883}{400}\pi\ mm^2$	22.6 mm^2	20.8 mm

VOLUMES OF SOLIDS

37.

	In terms of π	3 sig fig
a	$\dfrac{2000}{3}\pi\ cm^3$	2090 cm^3
b	$\dfrac{144}{125}\pi\ cm^3$	3.62 cm^3
c	$\dfrac{1125}{4}\pi\ cm^3$	884 cm^3
d	$\dfrac{29791}{750}\pi\ m^3$	125 m^3
e	$\dfrac{11979}{128}\pi\ m^3$	294 m^3
f	$7776\pi\ mm^3$	24400 mm^3

38.

	In terms of π	3 sig fig
a	$64\pi\ cm^3$	$201\ cm^3$
b	$\dfrac{4067}{250}\pi\ m^3$	$51.1\ m^3$
c	$1625\pi\ mm^3$	$5110\ mm^3$
d	$\dfrac{486}{125}\pi\ m^3$	$12.2\ m^3$
e	$\dfrac{84}{125}\pi\ m^3$	$2.11\ m^3$

39.

	In terms of π	3 sig fig
a	$21000\pi\ cm^3$	$66000\ cm^3$
b	$5832\pi\ mm^3$	$18300\ mm^3$
c	$\dfrac{375}{4}\pi\ cm^3$	$295\ cm^3$
d	$\dfrac{16762}{375}\pi\ m^3$	$140\ m^3$
e	$87000\pi\ cm^3$	$273000\ cm^3$
f	$\dfrac{4}{125}\pi\ m^3$	$0.101\ m^3$

40.

	Exact Value	3 sig fig
a	$\dfrac{200}{3} \ cm^3$	$66.7 \ cm^3$
b	$4840 \ m^3$	$4840 \ m^3$
c	$\dfrac{28577}{750} \ cm^3$	$38.1 \ cm^3$
d	$\dfrac{14080}{3} \ cm^3$	$4690 \ cm^3$
e	$\dfrac{833}{1875} \ cm^3$	$0.444 \ cm^3$
f	$\dfrac{183347}{3} \ mm^3$	$61100 \ mm^3$