Mathematics Higher

Assessment 4

Revision Materials

Integration, Logs & Exponentials, Recurrence Relations

Layout and content of the Unit Assessment will be different. This is not meant to be a carbon copy of the Unit Assessment. This booklet is an opportunity to practice all of the essential skills required to pass the Unit Assessment.

This booklet should be used to identify any areas for improvement **before** you sit the Unit assessment for the first time.

Unit	Assessment standard	Description
H4LD 76 **Relationships and Calculus**	1.4 Applying calculus skills of integration	♦ integrating an algebraic function which is, or can be, simplified to an expression of powers of x ♦ integrating functions of the form $f(x) = (x+q)^n$, $n \neq -1$ ♦ integrating functions of the form $f(x) = p\cos x$ and $f(x) = p\sin x$ ♦ calculating definite integrals of polynomial functions with integer limits
H4LC 76 **Expressions and Functions**	1.1 Applying algebraic skills to logarithms and exponentials	♦ simplifying a numerical expression, using the laws of logarithms and exponents ♦ solving logarithmic and exponential equations, using the laws of logarithms and exponents
H22J 76 **Applications**	1.4 Applying calculus skills to optimisation and area	♦ finding the area between a curve and the x-axis ♦ finding the area between two curves or a straight line and a curve
	1.3 Applying algebraic skills to sequences	♦ determining a recurrence relation from given information and using it to calculate a required term. ♦ finding and interpreting the limit of a sequence, where it exists

H4LD 76 Relationships and Calculus

1.4 Applying calculus skills of integration

- integrating an algebraic function which is, or can be, simplified to an expression of powers of *x*

Q1 Find:

a) $\int 3-\frac{2}{x^4}.dx,\ \ x\neq 0$ **b)** $\int 2x+\frac{3}{x^2}.dx,\ \ x\neq 0$ **c)** $\int \sqrt{x}+\frac{1}{x^3}.dx,\ \ x\neq 0$

d) $\int x(3+\frac{1}{x^5}).dx, x\neq 0$ **e)** $\int \frac{x}{x^3}+2x.dx,\ \ x\neq 0$ **f)** $\int \frac{5}{\sqrt[3]{x}}.dx,\ \ x\neq 0$

g) $\int 18x^2-\frac{1}{x^9}.dx, x\neq 0$ **h)** $\int 5+\frac{2}{x^3}.dx,\ \ x\neq 0$ **i)** $\int \frac{1}{x^2}-\frac{1}{x^3}.dx,\ \ x\neq 0$

j) $\int 3x-\frac{1}{\sqrt{x}}.dx,\ \ x\neq 0$ **j)** $\int \sqrt[3]{x}-\frac{1}{x^7}.dx,\ \ x\neq 0$ **k)** $\int \sqrt{x}\,(1+\frac{1}{x^3}).dx,\ \ x\neq 0$

H4LD 76 Relationships and Calculus

1.4 Applying calculus skills of integration

- integrating functions of the form $f(x) = p\cos x$ and $f(x) = p\sin x$

Q2 Find:

a) $\int 3\sin x\,.\,dx$ **b)** $\int 5\cos x\,.\,dx$ **c)** $\int -4\cos x\,.\,dx$

d) $\int \frac{4}{3}\sin x\,.\,dx$ **e)** $\int \frac{3}{7}\cos x\,.\,dx$ **f)** $\int 9\sin x\,.\,dx$

g) $\int \frac{2}{5}\sin x\,.\,dx$ **h)** $\int \sqrt{3}\cos x\,.\,dx$ **i)** $\int \frac{1}{\sqrt{2}}\sin x\,.\,dx$

j) $\int 2\cos\theta\,.\,d\theta$ **j)** $\int 7\sin r\,.\,dr$ **k)** $\int \frac{11}{7}\cos\omega\,.\,d\omega$

H4LD 76 Relationships and Calculus

1.4 Applying calculus skills of integration

- integrating functions of the form $f(x) = (x + q)^n$, $n \neq -1$
- calculating definite integrals of polynomial functions with integer limits

Q3 Evaluate:

a) $\int_0^2 4x\,.\,dx$ **b)** $\int_1^3 3x^2\,.\,dx$ **c)** $\int_1^2 4x^3\,.\,dx$

d) $\int_1^2 x\,.\,dx$ **e)** $\int_0^1 2x\,.\,dx$ **f)** $\int_{-1}^2 2x\,.\,dx$

g) $\int_0^2 3x\,.\,dx$ **h)** $\int_{-1}^1 4x\,.\,dx$ **i)** $\int_0^1 x^2\,.\,dx$

j) $\int_0^1 (4x + 3)\,.\,dx$ **j)** $\int_{-2}^2 (2x + 1)\,.\,dx$ **k)** $\int_0^2 (8 - 2x)\,.\,dx$

l) $\int_{-1}^1 (5 - 3x^2)\,.\,dx$ **m)** $\int_0^1 x^2 + 2x + 1\,.\,dx$ **n)** $\int_{-1}^1 x^2 + 2x + 1\,.\,dx$

o) $\int_0^2 (x + 1)(x - 1)\,.\,dx$ **p)** $\int_{-2}^0 (x - 1)^2\,.\,dx$ **q)** $\int_{-1}^1 3x^2 - 5x^4\,.\,dx$

r) $\int_{-1}^0 4t^3 + 3t^2\,.\,dt$ **s)** $\int_0^4 \sqrt{x}\,.\,dx$ **t)** $\int_1^2 \frac{dx}{x^2}$

u) $\int_1^4 \frac{dx}{\sqrt{x}}$ **v)** $\int_1^3 x^2 - \frac{1}{x^2}\,.\,dx$ **w)** $\int_1^2 2t + \frac{2}{t^3}\,.\,dt$

Q4 Evaluate

a) $\int_0^1 (x+4)^4 \, . dx$ **b)** $\int_1^2 (x-1)^3 \, . dx$ **c)** $\int_{-2}^{-1} (3x-2)^3 \, . dx$

d) $\int_{-1}^1 (5x-4)^3 \, . dx$ **e)** $\int_0^1 (4x-3)^6 \, . dx$ **f)** $\int_1^2 (5-2x)^{-3} \, . dx$

g) $\int_0^1 (2x+1)^3 \, . dx$ **h)** $\int_{-1}^1 (1-x)^3 \, . dx$ **i)** $\int_0^4 \sqrt{4-x} \, . dx$

j) $\int_1^2 \frac{dx}{(x+2)^2}$ **j)** $\int_1^2 \frac{dx}{\sqrt{2x+3}}$ **k)** $\int_{-1}^4 (3x+4)^{3/2} \, . dx$

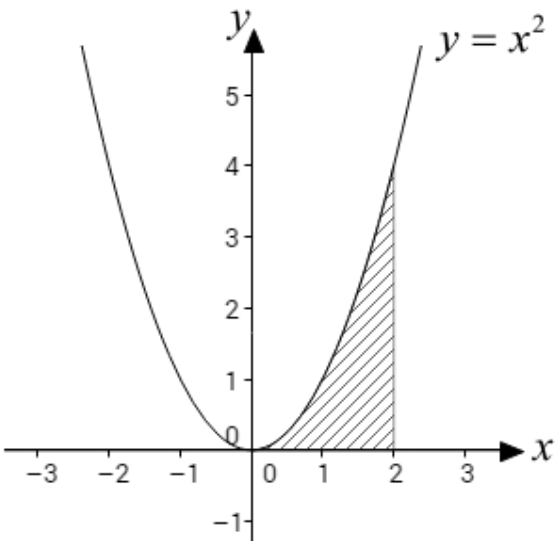
H22J 76 Applications

1.4 Applying calculus skills to optimisation and area

- finding the area between a curve and the *x*-axis

Q5 Find the area of the shaded sections for the following:

a)



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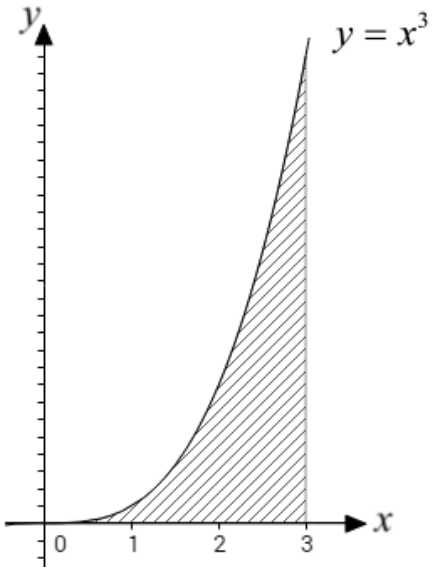

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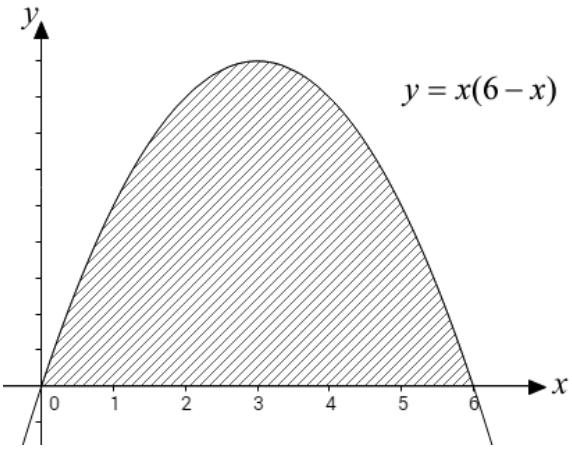

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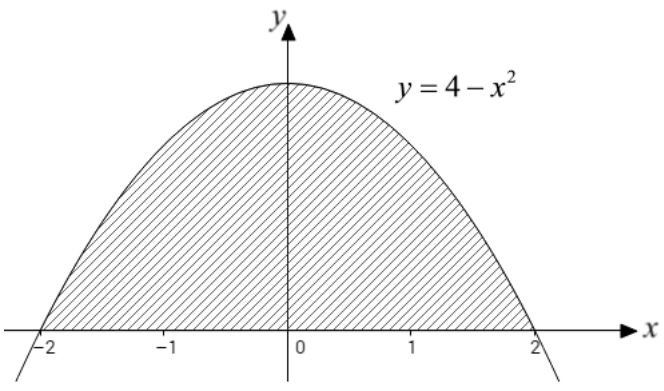
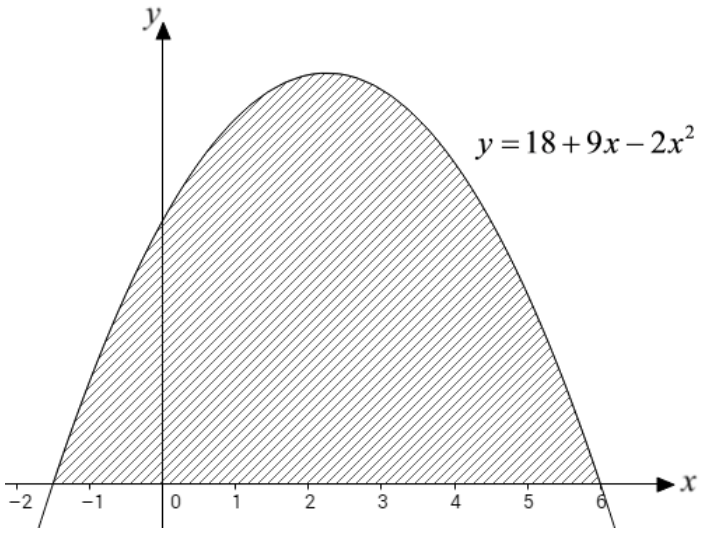
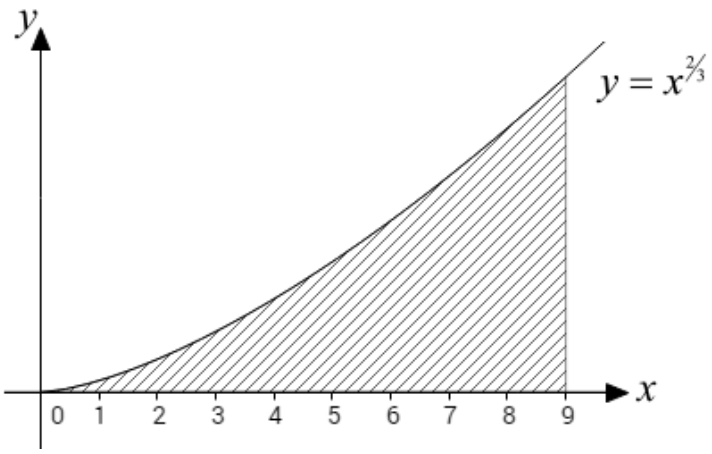


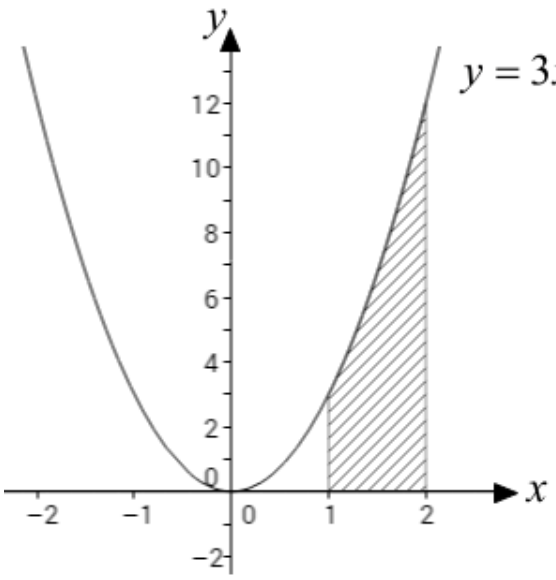
e)


y
x
y = 18 + 9x − 2x²
−2
−1
0
1
2
3
4
5
6

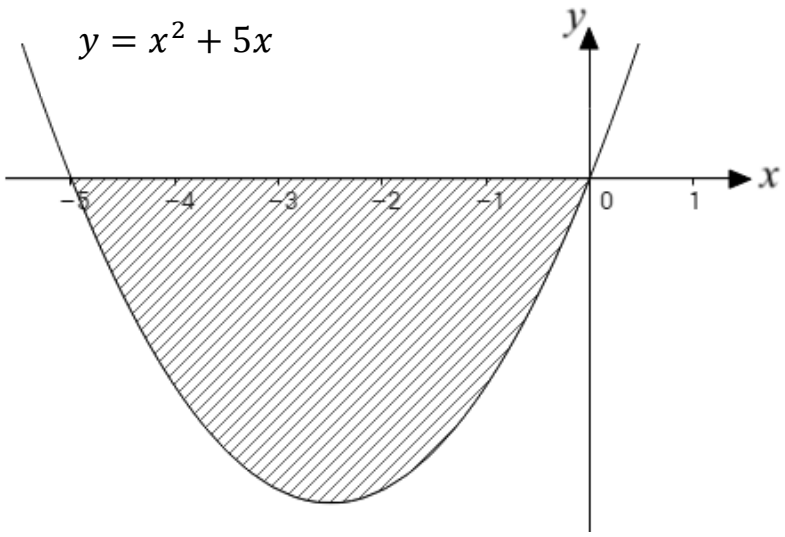
f)


y
x
y = x^(2/3)
0
1
2
3
4
5
6
7
8
9

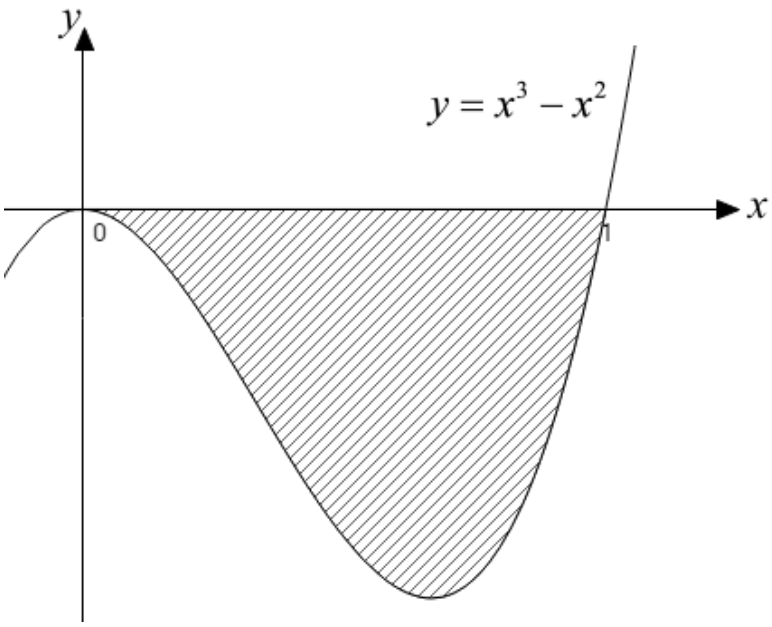
g)


y
x
y = 3x²
12
10
8
6
4
2
0
−2
−2
−1
0
1
2

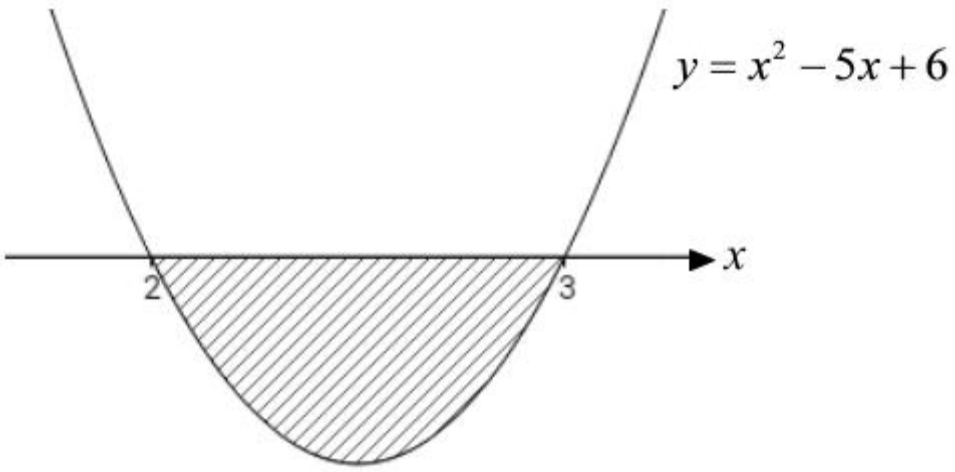
h)



i)



j)



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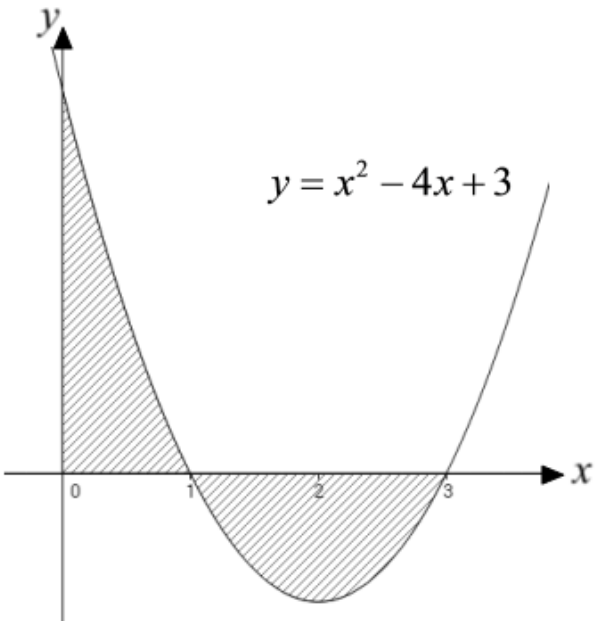
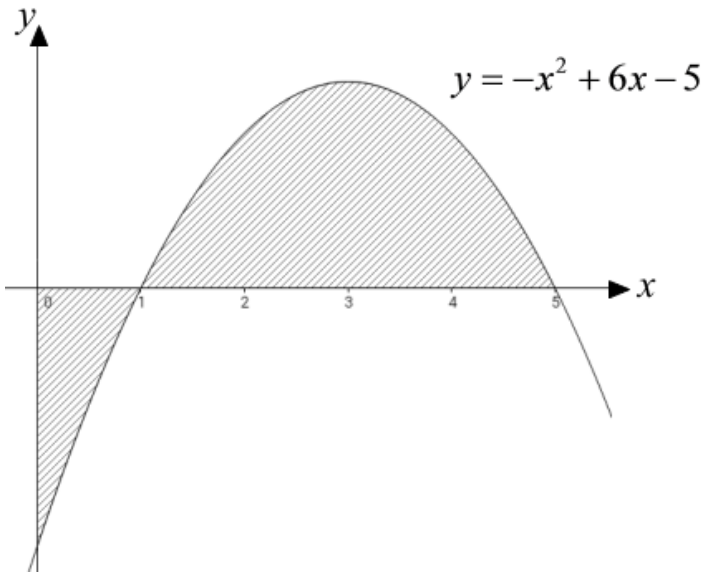
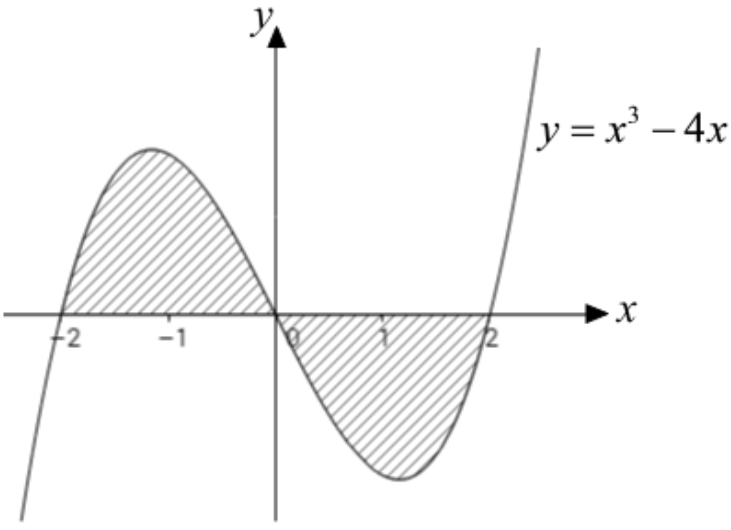


l)



m)



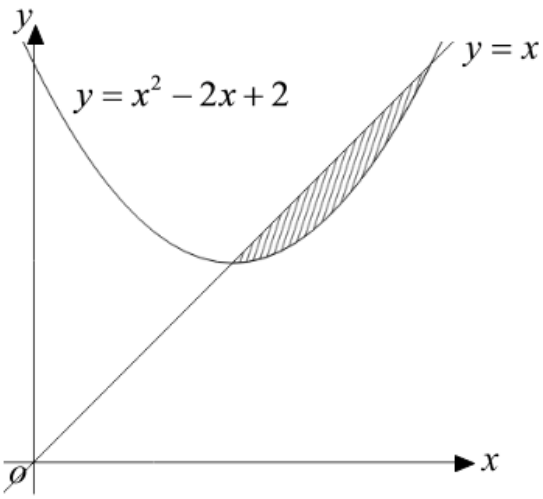
H22J 76 Applications

1.4 Applying calculus skills to optimisation and area

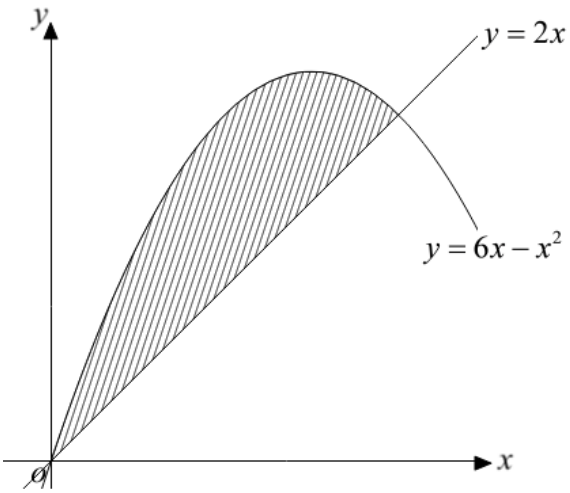
- finding the area between two curves or a straight line and a curve

Q6 Evaluate the shaded area for each of the following:

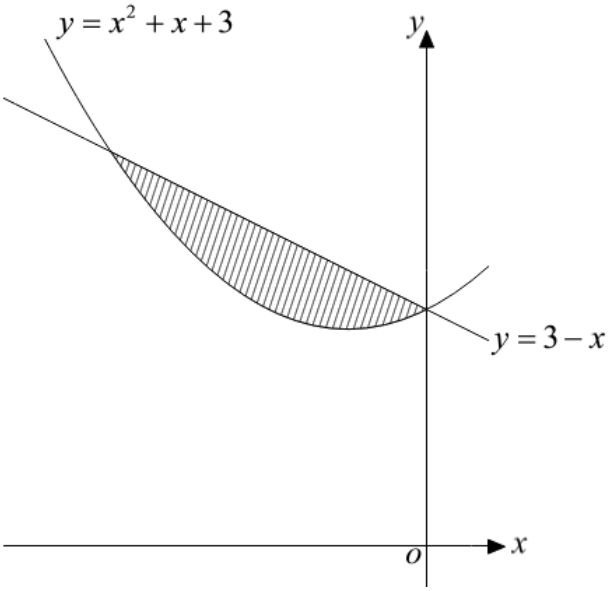
a)



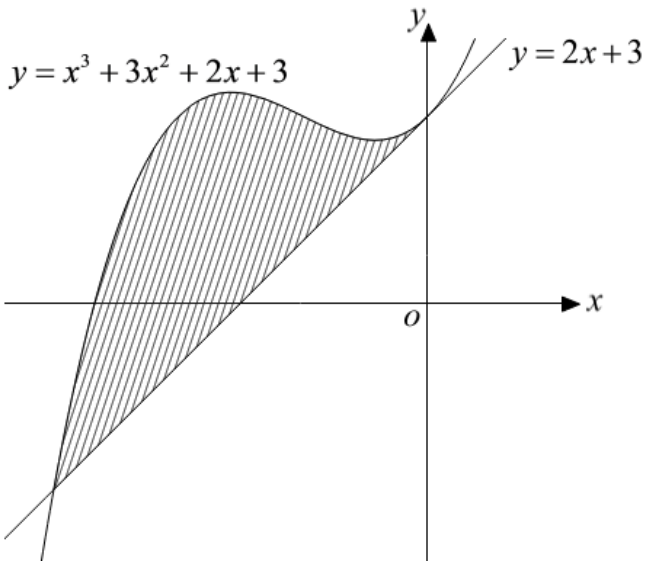
b)



c)



d)



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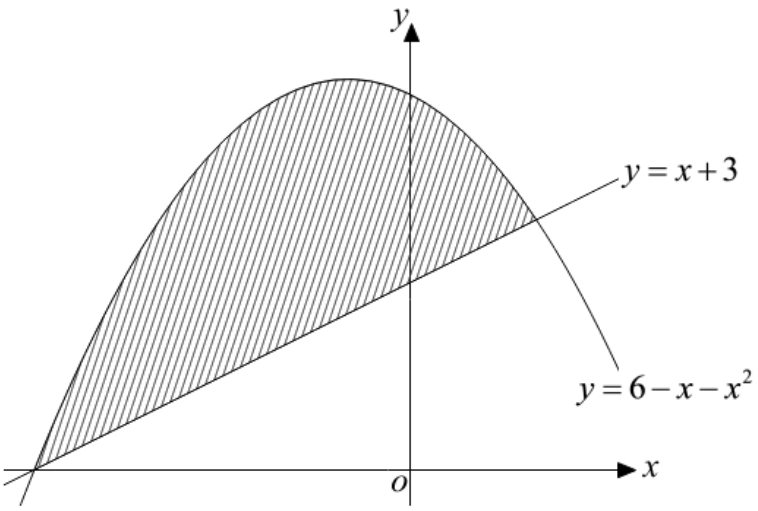
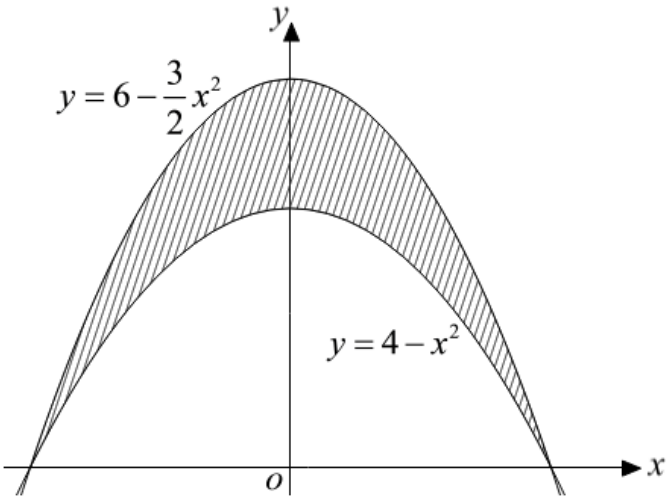
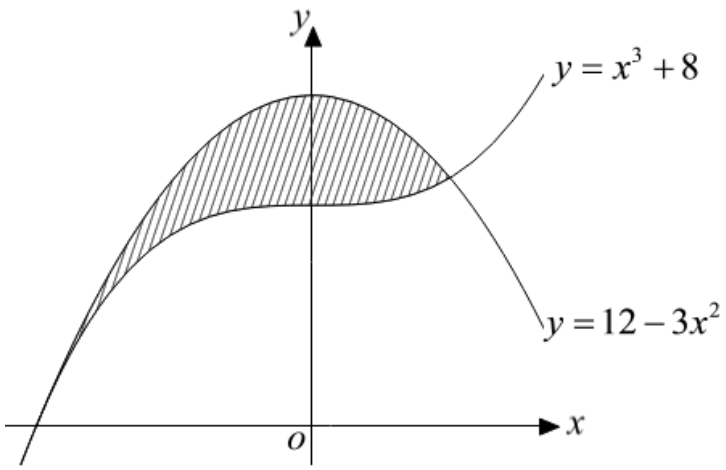


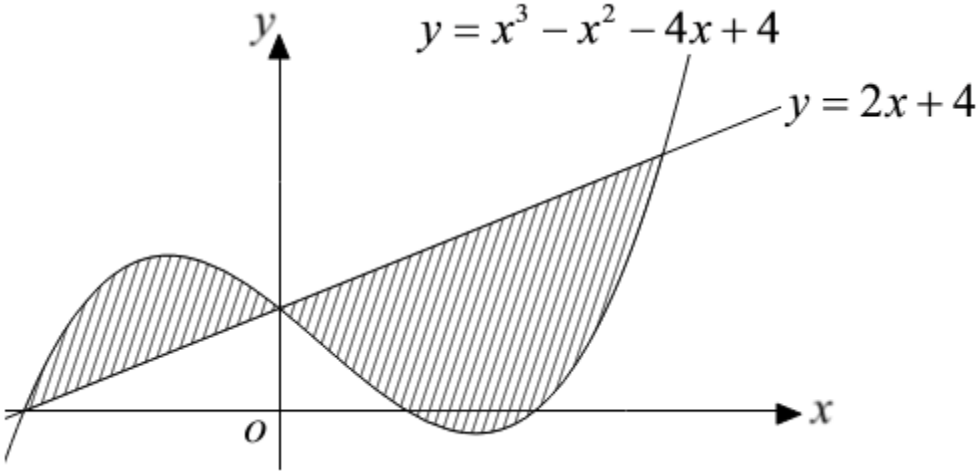
f)



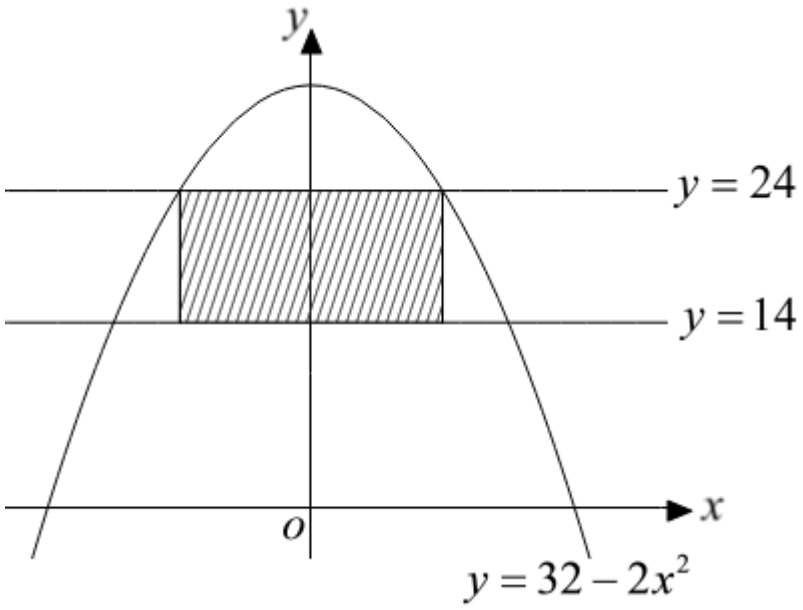
g)



h)



i)



H4LC 76 Expressions and Functions

1.1 Applying algebraic skills to logarithms and exponentials

- simplifying a numerical expression, using the laws of logarithms and exponents

Q7 Simplify the following:

a) $\log_a 7 + \log_a 2$ **b)** $\log_a 5 + \log_a 3$ **c)** $\log_a 10 + \log_a 1$

d) $\log_a 2 + \log_a 3$ **e)** $\log_a 12 - \log_a 2$ **f)** $\log_a 8 - \log_a 4$

g) $\log_a 27 + \log_a \frac{1}{3}$ **h)** $\log_a 5 - \log_a 25$ **i)** $\log_a 4 + \log_a 9 - \log_a 6$

j) $\log_a 3 + \log_a 8 - \log_a 4$ **j)** $\log_a 5 + \log_a 4 - \log_a 6$ **k)** $\log_a 9 + \log_a 6 - \log_a 4$

Q8 Simplify the following:

a) $\log_a 2 + 2\log_a 3$ **b)** $\log_a 4 + 3\log_a 2$

c) $\log_a 8 - 2\log_a 2$ **d)** $\log_a 25 - \log_a 5$

e) $\log_a 27 - 3\log_a 3$ **f)** $2\log_a 3 + 3\log_a 2$

g) $\log_a 64 - 2\log_a 4$ **h)** $5\log_a 2 - \log_a 4$

i) $\log_a 2 + 3\log_a 4$ **j)** $\log_a 12 - 2\log_a 2$

Q9 Simplify, then evaluate the following:

a) $\log_8 2 + \log_8 4$ **b)** $\log_6 2 + \log_6 3$

c) $\log_5 100 - \log_5 4$ **d)** $\log_4 18 - \log_4 9$

e) $2\log_2 4 - 3\ \log_2 2$ **f)** $2\log_6 2 + 2\ \log_6 3$

g) $2\log_{10} 5 + 2\log_{10} 2$ **h)** $3\log_3 3 + \frac{1}{2}\log_3 9$

i) $5\log_8 2 + \log_8 4 - \log_8 16$ **j)** $\log_2\left(\frac{1}{2}\right) - \log_2\left(\frac{1}{4}\right)$

H4LC 76 Expressions and Functions

1.1 Applying algebraic skills to logarithms and exponentials

- solving logarithmic and exponential equations, using the laws of logarithms and exponents

Q10 Solve the following:

a) $e^x = 8.2$ **b)** $e^x = 3.1$ **c)** $e^x = 7.6$ **d)** $e^x = 0.25$ **e)** $e^x = 7.2$

f) $e^x = 2.7$ **g)** $e^x = 2.12$ **h)** $e^x = 3.14$ **i)** $e^x = 5$ **j)** $e^x = 4.1$

k) $10^x = 102$ **l)** $10^x = 19$ **m)** $10^x = 73$ **n)** $10^x = 1050$ **o)** $10^x = 762$

p) $2^x = 0.3$ **q)** $1.2^x = \pi$ **r)** $0.7^x = 0.9$ **s)** $4^x = 25$ **t)** $1.5^x = 7.5$

H22J 76 Applications

1.3 Applying algebraic skills to sequences

- finding and interpreting the limit of a sequence, where it exists

Q11 For each of these recurrence relationships

i. List the values of $U_1, U_2 and\ U_3$
ii. Say whether or not a limit exists, giving a reason.
iii. Calculate the limit, if it exists.

a) $U_{n+1} = 0.5\,U_n + 5 \ \ and \ \ U_0 = 6$ **b)** $U_{n+1} = 1.5\,U_n + 1 \ \ and \ \ U_0 = 16$

c) $U_{n+1} = -0.4\,U_n + 30 \ \ and \ \ U_0 = 38$ **d)** $U_{n+1} = 1.1U_n + 4 \ and \ \ U_0 = 5.1$

e) $U_{n+1} = 0.9U_n + 3 \ \ and \ \ U_0 = 48$ **f)** $U_{n+1} = -0.8U_n + 10 \ \ and \ \ U_0 = 30$

g) $U_{n+1} = 0.6\,U_n - 4 \ \ and \ \ U_0 = -1$ **h)** $U_{n+1} = -4U_n - 2 \ \ and \ \ U_0 = 38$

i) $U_{n+1} = \frac{2}{3}U_n + 7 \ \ and \ \ U_0 = 3$ **j)** $U_{n+1} = 0.25\,U_n + 72 \ \ and \ \ U_0 = 8$

H22J 76 Applications

1.3 Applying algebraic skills to sequences

- determining a recurrence relation from given information and using it to calculate a required term.
- finding and interpreting the limit of a sequence, where it exists

Q12 A mushroom bed has 1000 mushrooms ready for picking. Each morning 70% of the crop are picked, and each night another 300 are ready for picking.

a) Write down a recurrence relation.

b) How many are ready for picking after 3 days?

c) Calculate the limit of the sequence.

Q13 A scientist is studying a large flock of 200 birds. Every minute 10% of the birds leave the flock and 30 birds return.

a) Write down a recurrence relation.

b) How many birds are in the flock after 5 minutes?

c) What can be said about the size of the flock in the long run?

Q14 At present, just after high tide, a harbour has 1 metre of silt. A dredger removes 70% of the silt between high tides, but each high tide deposits an extra 1.2 metres of silt. A depth of silt greater than 1.8 metres will make the harbour unusable.

a) Write down a recurrence relation.

b) Find the depth of the silt to the nearest cm after 4 high tides.

c) Will the harbour become unusable? Give a reason for your answer.

Q15 For safety reasons a tyre must have a pressure of over 25 Units and under 28 Units. A slow puncture makes a tyre lose 11% of its pressure every day, but every morning, before use, 3 Units of pressure are put in the tyre. Will the pressure remain within safe limits if it starts at 26 Units? Give a reason for your answer.

Q16 A pond is treated weekly with a chemical to ensure that the number of bacteria is kept low. It is estimated that the chemical kills 68% of all bacteria. Between the weekly treatments, it is estimated that 60 million new bacteria appear.

There are u_n million bacteria at the start of a particular week.

a) Write down the recurrence relation for U_{n+1} , the number of millions of bacteria at the start of the next week.

b) If the level of bacteria in the pond exceeds 100 million then the plant life in the pond will start to die.

i. Find the limit of the of the sequence generated by this recurrence relation as $n \to \infty$

ii. In the long term will the pond be able to sustain its plant life?

Q17 In a small colony, 20% of the existing insects are eaten by predators each day; however during the night 400 insects are hatched.

There are U_n insects at the start of a particular day.

a) Write down the recurrence relation for U_{n+1} , the number of insects at the start of the next day.

b) If the number of insects in the colony exceeds 2500 there will not be enough food to sustain the colony in its current location and the colony will have to migrate to a new area.

i. Find the limit of the of the sequence generated by this recurrence relation as $n \to \infty$

ii. In the long term what will this mean for the colony?

Q18 In a small rabbit colony one eighth of all existing rabbits are eaten by predators each summer, however over the winter a further 24 rabbits are born.

There are u_n rabbits at the start of a particular summer.

a) Write down the recurrence relation for U_{n+1} , the number of rabbits at the start of the next summer.

b) If the number of rabbits exceeds 150 there will not be enough room in the existing warrens and there will be a need to split the rabbit colony into two smaller colonies.

i. Find the limit of the of the sequence generated by this recurrence relation as $n \to \infty$

ii. In the long term what will this mean for the rabbit colony?

ANSWERS

Q1

a)	$3x + \frac{2}{3}x^{-3} + c$	**b)**	$x^2 - 3x^{-1} + c$	**c)**	$2x^{-\frac{1}{2}} - \frac{1}{2}x^{-2} + c$
d)	$\frac{3}{2}x^2 - \frac{1}{3}x^{-3} + c$	**e)**	$-x^{-1} + x^2 + c$	**f)**	$\frac{15}{2}x^{\frac{2}{3}} + c$
g)	$6x^3 + \frac{1}{8}x^{-8} + c$	**h)**	$5x - x^{-2} + c$	**i)**	$-x^{-1} + \frac{1}{2}x^{-2} + c$
j)	$\frac{3}{2}x^2 - \frac{1}{3}x^{-3} + c$	**j)**	$\frac{3}{4}x^{\frac{4}{3}} + \frac{1}{6}x^{-6} + c$	**k)**	$\frac{2}{3}x^{\frac{3}{2}} - \frac{2}{3}x^{-\frac{3}{2}} + c$

Q2

a)	$-3\cos x + c$	**b)**	$5\sin x + c$	**c)**	$-4\sin x + c$
d)	$-\frac{4}{3}\cos x + c$	**e)**	$\frac{3}{7}\sin x + c$	**f)**	$-9\cos x + c$
g)	$-\frac{2}{5}\cos x + c$	**h)**	$\sqrt{3}\sin x + c$	**i)**	$-\frac{1}{\sqrt{2}}\cos x + c$
j)	$2\sin\theta + c$	**j)**	$-7\cos r + c$	**k)**	$\frac{11}{7}\sin\omega + c$

Q3

a)	8	**b)**	26	**c)**	15
d)	$\frac{3}{2}$	**e)**	1	**f)**	3
g)	6	**h)**	0	**i)**	$\frac{1}{3}$
j)	5	**j)**	4	**k)**	12
l)	8	**m)**	$\frac{7}{3}$	**n)**	$\frac{8}{3}$
o)	$\frac{2}{3}$	**p)**	$\frac{26}{3}$	**q)**	0
r)	0	**s)**	$\frac{16}{3}$	**t)**	$\frac{1}{2}$
u)	2	**v)**	8	**w)**	$\frac{15}{4}$

Q4

a)	$\frac{2101}{5}$	**b)**	$\frac{1}{4}$	**c)**	$-\frac{1157}{4}$
d)	-328	**e)**	$\frac{547}{7}$	**f)**	$\frac{2}{9}$
g)	10	**h)**	4	**i)**	$\frac{16}{3}$
j)	$\frac{1}{12}$	**j)**	$\sqrt{7}-\sqrt{5}$	**k)**	$\frac{682}{5}$

Q5

a)	$\frac{7}{3}$	**b)**	20	**c)**	36
d)	$\frac{32}{3}$	**e)**	$\frac{1125}{8}$	**f)**	$\frac{9^{\frac{13}{6}}}{5} \approx 23.36$
g)	7	**h)**	$\frac{125}{6}$	**i)**	$\frac{1}{12}$
j)	$\frac{1}{6}$	**k)**	$\frac{8}{3}$	**l)**	13
m)	8				

Q6

a)	$\frac{1}{6}$	**b)**	$\frac{32}{3}$	**c)**	$\frac{4}{3}$
d)	$\frac{27}{4}$	**e)**	$\frac{32}{3}$	**f)**	$\frac{16}{3}$
g)	$\frac{27}{4}$	**h)**	$\frac{253}{12}$	**i)**	40

Q7

a)	$\log_a 14$	**b)**	$\log_a 15$	**c)**	$\log_a 10$
d)	$\log_a 6$	**e)**	$\log_a 6$	**f)**	$\log_a 2$
g)	$\log_a 9$	**h)**	$\log_a \frac{1}{5}$	**i)**	$\log_a 6$
j)	$\log_a 6$	**j)**	$\log_a \frac{10}{3}$	**k)**	$\log_a \frac{27}{2}$

Q8

a) $\log_a 18$ **b)** $\log_a 32$ **c)** $\log_a 2$

d) $\log_a 5$ **e)** 0 **f)** $\log_a 72$

g) $\log_a 4$ **h)** $\log_a 8$ **i)** $\log_a 128$

j) $\log_a 3$

Q9

a) $\log_8 8 = 1$ **b)** $\log_6 6 = 1$ **c)** $2\log_5 5 = 2$

d) $\frac{1}{2}\log_4 4 = \frac{1}{2}$ **e)** $\log_2 1 = 0$ **f)** $2\log_6 6 =$

g) $2\log_{10} 10 = 2$ **h)** $4\log_3 3 = 4$ **i)** $\log_8 8 = 1$

j) $\log_2 2 = 1$

Q10

a) $\ln 8.2 = 2.104\ldots$ **b)** $\ln 3.1 = 1.1314\ldots$ **c)** $\ln 7.6 = 2.02814\ldots$

d) $\ln 0.25 = -1.0386\ldots$ **e)** $\ln 7.2 = 1.9740\ldots$ **f)** $\ln 2.7 = 0.9932\ldots$

g) $\ln 2.12 = 0.7514\ldots$ **h)** $\ln 3.14 = 1.1442\ldots$ **i)** $\ln 5 = 1.6094\ldots$

j) $\ln 4.1 = 1.4109\ldots$ **k)** $\log_{10} 102 = 2.0086\ldots$ **l)** $\log_{10} 19 = 1.2787\ldots$

m) $\log_{10} 73 = 1.8633\ldots$ **n)** $\log_{10} 1050 = 3.0211\ldots$ **o)** $\log_{10} 762 = 2.8819\ldots$

p) $\frac{\ln 0.3}{\ln 2} = -1.7369\ldots$ **q)** $\frac{\ln \pi}{\ln 1.2} = 6.2786\ldots$ **r)** $\frac{\ln 0.9}{\ln 0.7} = 0.2953\ldots$

s) $\frac{\ln 25}{\ln 4} = 2.3219\ldots$ **t)** $\frac{\ln 7.5}{\ln 1.5} = 4.9693\ldots$

Q11

a) **i)**

$U_1 = 8,$

$U_2 = 9,$

$U_3 = 9.5$

ii)

$Yes, -1 < 0.5 < 1$

iii)

$\lim_{n\to\infty} = 10$

b) **i)**

$U_1 = 25$

$U_2 = 38.5$

$U_3 = 58.75$

ii)

$NO\ Limit$

iii)

N/A

c) **i)**

$U_1 = 14.8$

$U_2 = 24.08$

$U_3 = -66.32$

ii)

$Yes, -1 < -0.4 < 1$

iii)

$\lim_{n\to\infty} = \frac{150}{7}$

d) **i)**

$U_1 = 9.61$

$U_2 = 14.571$

$U_3 = 20.0281$

ii)

$NO\ Limit$

iii)

N/A

e) **i)**

$U_1 = 46.2$

$U_2 = 44.58$

$U_3 = 43.122$

ii)

$Yes, -1 < 0.9 < 1$

iii)

$\lim_{n\to\infty} = 30$

f) **i)**

$U_1 = -14$

$U_2 = 21.2$

$U_3 = -13.96$

ii)

$Yes, -1 < -0.8 < 1$

iii)

$\lim_{n\to\infty} = \frac{50}{9}$

g) **i)**

$U_1 = -4.6$

$U_2 = -6.76$

$U_3 = -8.056$

ii)

$Yes, -1 < 0.6 < 1$

iii)

$\lim_{n\to\infty} = -10$

h) **i)**

$U_1 = -154$

$U_2 = 614$

$U_3 = -2458$

ii)

$NO\ Limit$

iii)

N/A

i) **i)**

$U_1 = 9$

$U_2 = 13$

$U_3 = 15\frac{2}{3}$

ii)

$Yes, -1 < \frac{2}{3} < 1$

iii)

$\lim_{n\to\infty} = 21$

j) **i)**

$U_1 = 74$

$U_2 = 90.5$

$U_3 = 94.625$

ii)

$Yes, -1 < 0.25 < 1$

iii)

$\lim_{n\to\infty} = 96$

Q12

a) $U_{n+1} = 0.3\,U_n + 300$

b) $U_3 = 444$

c) $Lim = \frac{3000}{7} \approx 428.6$

Q13

a) $U_{n+1} = 0.9\,U_n + 30$

b) $U_5 \approx 836$

c) $Lim = 300$. The flock will have a maximum of 300 birds.

Q14

a) $U_{n+1} = 0.3\,U_n + 1.2$

b) $U_4 = 1.7085$

c) $Lim = \frac{12}{7} \approx 1.714.$ The harbour will never have more than 1.8 metres so will never be unusable.

Q15 $Lim = \frac{300}{11} \approx 27.3$ No the tyre will remain within safe limits. The limit of 27.3 is the upper limit. The lower limit is 24.3. This is out with the safe limits for the tyre.

Q16

a) $U_{n+1} = 0.32\,U_n + 60$

b)

i. $Lim = \frac{1500}{17} \approx 88.24\ million$

ii. Yes the pond will be able to sustain its plant life as the upper limit is below 100 million.

Q17

a) $U_{n+1} = 0.8\,U_n + 400$

b)

i. $Lim = 2000$

ii. The colony will be safe as the upper limit is less than 2500.

Q18

a) $U_{n+1} = \frac{7}{8}\,U_n + 24$

b)

i. $Lim = 192$

ii. The rabbit colony will have to split in to 2 smaller colonies as the limit is higher than 150.