Higher Maths - Expressions and Formulae Revision Questions

Outcome 1.1 Applying algebraic skills to logarithms and exponentials

1. Simplify fully

(a) $\log_4 2 + \log_4 8$ (b) $\log_3 108 - \log_3 4$ (c) $\log_3 18 - \log_3 2$

(d) $\log_5 100 - \log_5 4$ (e) $\log_4 8 + \log_4 8$ (f) $2\log_{10} 2 + 2\log_{10} 5$

(g) $\log_9 3 - \log_9 6 + \log_9 18$ (h) $\log_3 9 - \log_3 \frac{1}{3}$ (i) $\frac{1}{2}\log_2 16 - \frac{1}{3}\log_2 8$

2. Solve the logarithmic equations for $x > 0$

(a) $\log_4(x + 3) = 2$ (b) $\log_3(x - 2) = 4$ (c) $\log_6(x - 8) = 2$

(d) $\log_a 4 + \log_a x = \log_a 12$ (e) $2\log_a 3 + \log_a x = \log_a 36$ (f) $\log_a(2x + 1) + \log_a 3x = \log_a 63$

(g) $\log_2 x + \log_2(x - 3) = 2$ (h) $\log_2(x - 1) + \log_2(x + 1) = 3$ (i) $\log_3 6x - \log_3(x - 2) = 2$

3. Given $2\log_m n = \log_m 16 + 1$, show that $n = 4\sqrt{m}$

4. The mass, M grams, of a radioactive isotope after a time of t years, is given by the formula

$M = M_o e^{-kt}$ where M_o is the initial mass of the isotope.

In 5 years a mass of 10 grams of the isotope is reduced to 8 grams.

(a) Calculate k.

(b) Calculate the half-life of the substance i.e. the time taken for half the substance to decay.

5. A cell culture grows at a rate given by the formula $y(t) = Ae^{kt}$ where A is the initial number of cells and y(t) is the number of cells after t hours.

(a) It takes 24 hours for 500 cells to increase in number to 800. Find k.

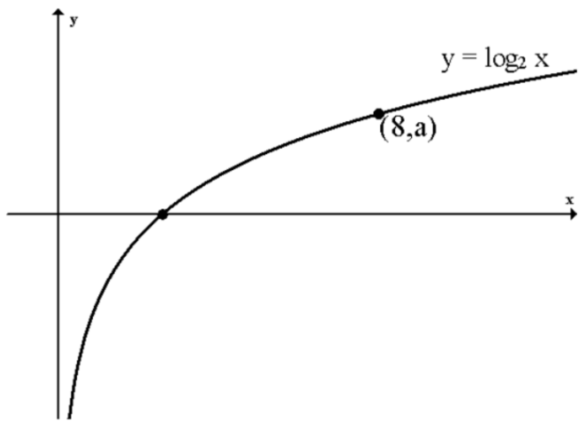
(b) Calculate the time taken for the number of cells to double.

6. The diagram opposite shows the graph of y = log2 x.

(a) Find the value of a.

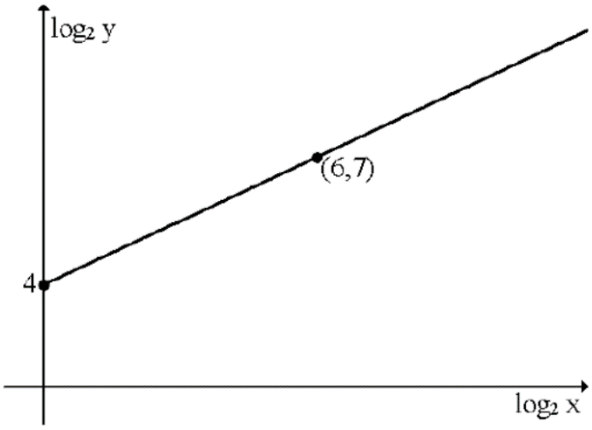
(b) Sketch the graph of $y = \log_2 x - 3$

(c) Sketch the graph of $y = \log_2 4x$



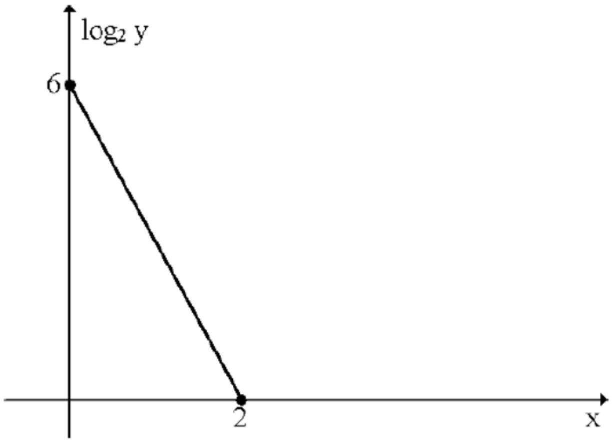
7. The graph opposite illustrates the law $y = kx^n$.

Find the values of k and n.



8. The graph opposite illustrates the law $y = ab^x$.

Find the values of a and b.



9. The concentration of the pesticide, *Xpesto*, in soil can be modelled by the equation $P_t = P_0 e^{-kt}$,

where:

- P_0 is the initial concentration;
- P_t is the concentration at time t;
- t is the time, in days, after the application of the pesticide.

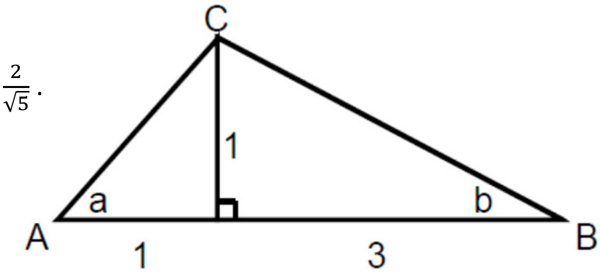
(*a*) Once in the soil, the half-life of a pesticide is the time taken for its concentration to be reduced to one half of its initial value.

If the half-life of *Xpesto* is 25 days, find the value of k to 2 significant figures.

(*b*) Eighty days after the initial application, what is the percentage decrease in concentration of *Xpesto*?

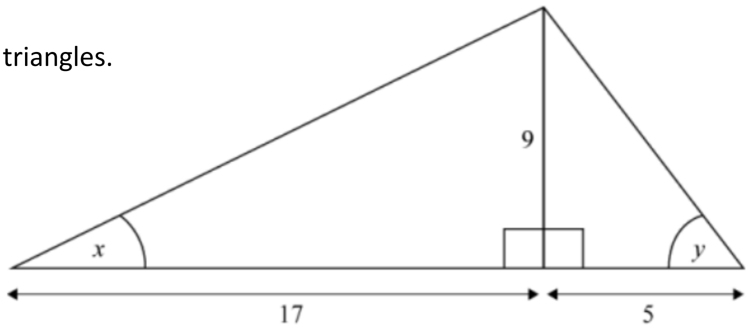
Outcome 1.2 Applying trigonometric skills to manipulating expressions

1. In triangle ABC, show that the exact value of sin(a + b) is $\frac{2}{\sqrt{5}}$.



2. The diagram shows two right-angled triangles.

   Find the exact value of sin (x-y).



3. Show that (3 + 2 cos x) (3 – 2 cos x) = 4 $\sin^2$x + 5.

4. Show that (3 + 2 cos x)(3 – 2 cos x) = 5 + 4 $\sin^2$x.

5. Express 2sinx + 3cosx in the form ksin(x + a)^0 where k>0 and $0 \leq x \leq 360$.

   Calculate the values of k and a

6. Express cos x – sin x in the form kcos(x - α), where k > 0 and 0 ≤ α ≤ 360.

7. (*a*) The expression can be written in the form *k* sin(*x* – *a*)°, where *k* >0 and 0 ≤ *a* < 360.

   Calculate the values of *k* and *a*.

   (*b*) Determine the maximum value of , where 0 ≤ *x* < 360.

8. Solve 4sin x + 3cos x = 2.5, $0 \le x \le 180$.

9. (*a*) Diagram 1 shows a right angled triangle, where the line OA has equation $3x - 2y = 0$.

(i) Show that $\tan a = \frac{3}{2}$.

(ii) Find the value of sin*a.*

(*b*) A second right angled triangle is added as shown in Diagram 2.

The line OB has equation $3x - 4y = 0$.

Find the values of sin*b* and cos*b.*

(*c*) (i) Find the value of $\sin(a - b)$.

(ii) State the value of $\sin(b - a)$.

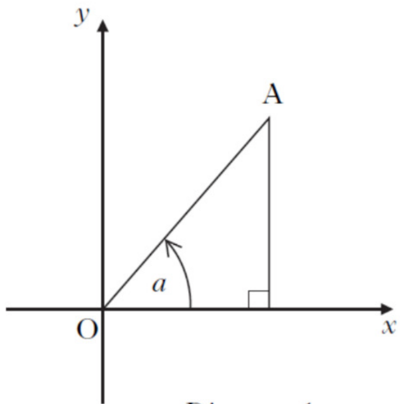


Diagram 1

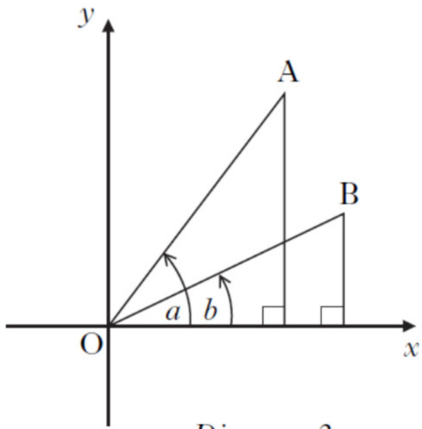


Diagram 2

Outcome 1.3 Applying algebraic and trigonometric skills to functions

1. Sketch the graph of $y = 3\cos(x + \frac{\pi}{4})$ for $0 \le x \le 2\pi$.

Show clearly the intercepts on the x-axis and the coordinates of the turning points.

2. The diagram shows the graph of y = f (x) with a maximum turning point at (–2, 3) and a minimum turning point at (1, –2).

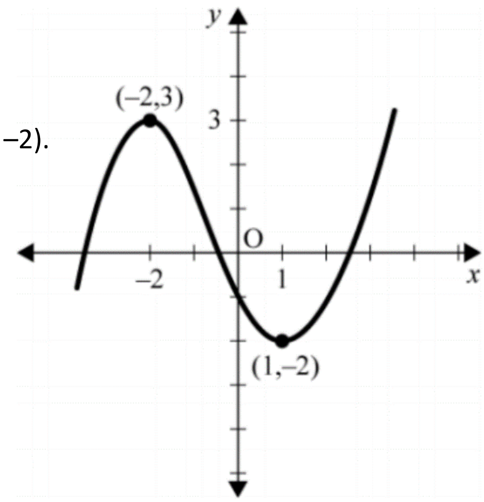
Sketch the graph of

(a) y = f(x – 2) – 3.

(b) y = f(x + 4) + 1.

(c) y = 2f(x) + 3

(d) y = f(2x) - 2



3. Sketch the following graphs

(a) $y = 4\sin x – 1$ $\quad 0 \le x \le 360$

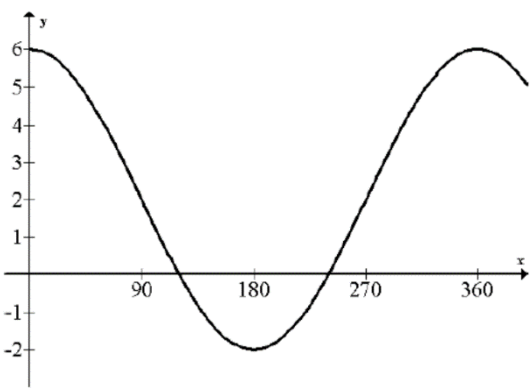
(b) $y = 4\cos 3x + 1$ $\quad 0 \le x \le 180$

(c) $y = 2\sin(x – 40)$ $\quad 0 \le x \le 2\pi$

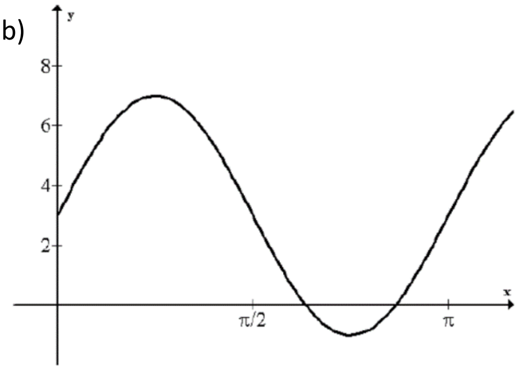
(d) $y = 3\cos(2x + 30) – 1$ $\quad 0 \le x \le \pi$

4. Write down the equation of each graph below in the form y = asinbx + c or y = acosbx + c.

(a)



(b)

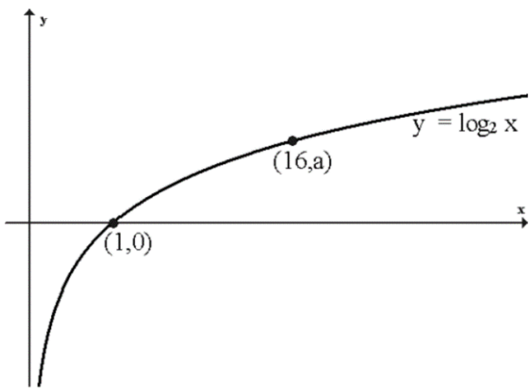


5. The diagram shows part of the graph of $y = \log_2 x$.

(a) Find the value of a.

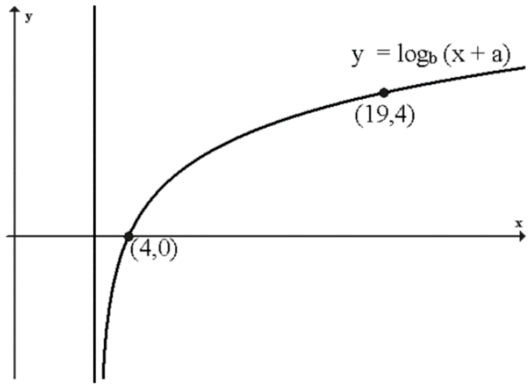
(b) Sketch the graph of $y = \log_2 x – 4$.

(c) Sketch the graph of $y = \log_2 8x$.



6. The diagram shows the graph of $y = \log_b (x + a)$.

Find the values of a and b.



7. $f(x) = 2x^2$ and $g(x) = 5x – 4$.

(a) Find f(g(2)).

(b) Find a formula for f(g(x)).

8. $f(x) = (x – 1)(x + 3)$ and $g(x) = x^2 + 3$.

Show that $f(g(x)) – g(g(x)) = 2x^2$.

9. The functions f and g, defined on suitable domains, are given by

$f(x) = \frac{1}{x^2-4}$ and $g(x) = x + 1$

(a) Find an expression for h(x), where h(x) = f(g(x)).

Give your answer as a single fraction.

(b) State a suitable domain for h.

10. $f(x) = 3x – 2$ and $g(x) = 3x + 2$

(a) Find formulae for f(g(x)) and g(f(x)).

(b) Find the least value of the product f(g(x)) × g(f(x)).

11. Write down the inverse function

(a) $f(x) = 4x - 5$ (b) $f(x) = \frac{x}{6}$ (c) $f(x) = \frac{2x}{5} + 4$ (d) $f(x) = \frac{2x-5}{4}$

(e) $f(x) = \frac{4x+7}{2}$ (f) $f(x) = 12 - \frac{3}{4}x$ (g) $f(x) = \frac{8-3x}{13}$ (h) $f(x) = \frac{-3x+4}{-9}$

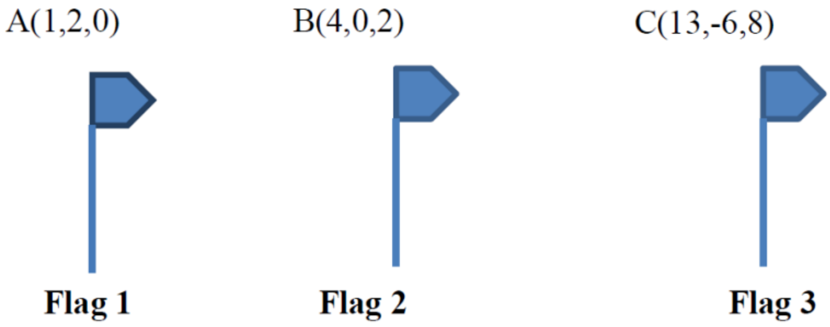
Outcome 1.4 Applying geometric skills to vectors

1. An engineer laying flags needs to check that:

   · they are in a straight line;

   · the distance between Flag 2 and Flag 3 is 3 times the distance between Flag 1 and Flag 2.

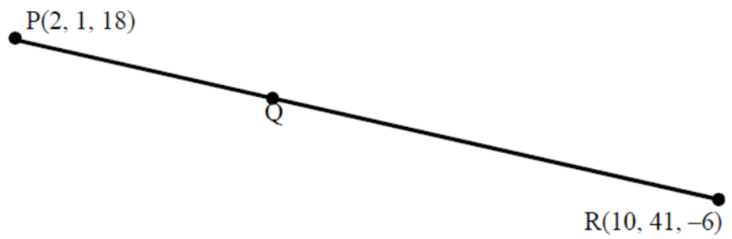
   Relative to suitable axes, the top-left corner of each flag can be represented by the points A (1, 2, 0), B (4, 0, 2), and C (13,–6, 8) respectively. All three flags point vertically upwards.



   Do the three flags meet the conditions given?

2. The points P, Q and R lie in a straight line, as shown. Q divides PR in the ratio 3:5.

   Find the coordinates of the point Q.



3. A is (0,-3,5), B is (7,-6,9) and C is (21,-12,17). Show that A, B and C are collinear stating the ratio AB:BC.

4. **u** = 2**i** – 2**j** + 4**k** and **v** = **i** + a**j** + 7**k**. If $|\boldsymbol{u}| = |\boldsymbol{v}|$find the value of a.

5. A triangle has vertices A(6,-1,9), B(3,-2,11) and C(7,-8,14). Show that this triangle is right-angled at B.

6. A triangle is formed from R(0,4,-1), S(1,5,2) and T(6,1,-2).

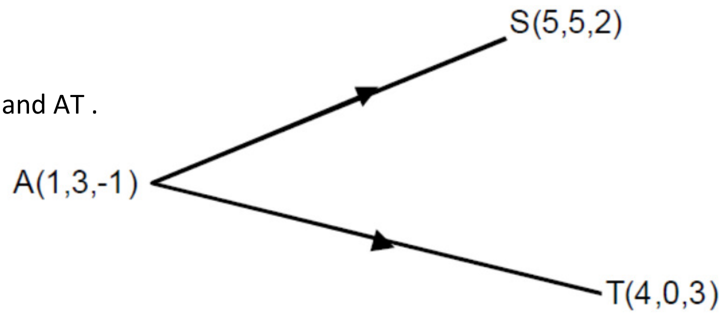
(a) Find the vectors RS and RT.

(b) Evaluate RS .RT

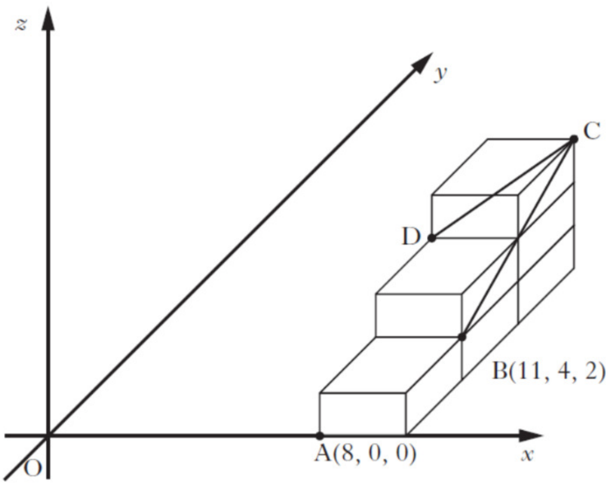
(c) What can you deduce about the lines RS and RT?

7. (a) For the diagram opposite find AS and AT .

(b) Hence calculate angle TAS.



8. Six identical cuboids are placed with their edges parallel to the coordinate axes as shown in the diagram.



A and B are the points (8, 0, 0) and (11, 4, 2) respectively.

(*a*) State the coordinates of C and D.

(*b*) Determine the components of $\overrightarrow{CB}$ and $\overrightarrow{CD}$.

(*c*) Find the size of the angle BCD.

9. (*a*) (i) Show that the points A(–7, –8, 1), T(3, 2, 5) and B(18, 17, 11) are collinear.

(ii) Find the ratio in which T divides AB.

(*b*) The point C lies on the *x*-axis.

If $\overrightarrow{TB}$ and $\overrightarrow{TC}$ are perpendicular, find the coordinates of C.

10. A is the point (3, –3, 0), B is (2, –3, 1) and C is (4, k, 0).

(a) (i) Express $\overrightarrow{BA}$ and $\overrightarrow{BC}$ in component form.

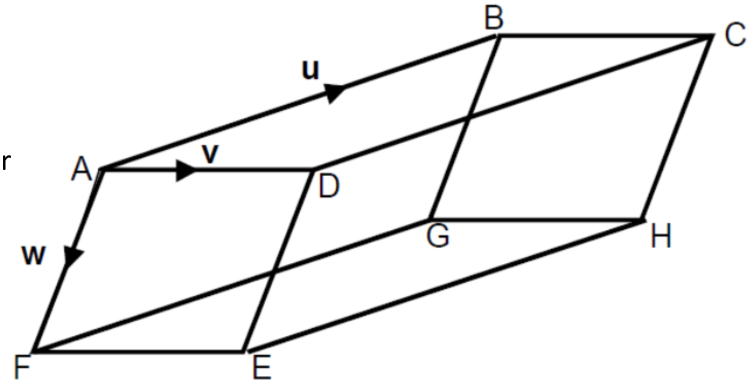
(ii) Show that cos $A\hat{B}C = \frac{3}{\sqrt{2(k^2+6k+14)}}$

(b) If angle ABC = 30 °, find the possible values of k.

11. ABCDEFGH is a parallelepiped.

In terms of u, v and w find expressions for

(a) $\overrightarrow{DC}$ (b) $\overrightarrow{HC}$ (c) $\overrightarrow{AC}$ (d) $\overrightarrow{FD}$ (e) $\overrightarrow{CF}$



12. The diagram shows the circles with equations

$(x + 2)^2 + (y + 4)^2 = 100$

and

$x^2 + y^2 – 20x – 10y + 100 = 0$

Find the coordinates of the point P.