Balerno Mathematics Course - Higher

1. Topic Skill	Subskill	Support notes (from SQA)
A1.1 **Straight line:** Applying algebraic skills to rectilinear shapes	♦ Finding the equation of a line parallel to and a line perpendicular to a given line	Emphasise the 'gradient properties' of $m_1 = m_2$ and $m_1 m_2 = -1$ Use practical contexts for triangle work where possible. Emphasise differences in median, altitude etc. Perhaps investigate properties and intersections. Avoid approximating gradients to decimals. Knowledge of the basic properties of triangles and quadrilaterals would be useful. In order to 'show' collinearity, statement should include mention of 'common point', eg 'since $m_{AB} = m_{BC}$ and B is a common point.' Understanding of terms such as orthocentre, circumcentre and concurrency.
	♦ Using $m = \tan\theta$ to calculate a gradient or angle	
	➢ Using properties of medians, altitudes and perpendicular bisectors in problems involving the equation of a line and intersection of lines	
	➢ Determine whether or not two lines are perpendicular	

2. Topic Skill	Subskill	Support notes (from SQA)
R1.1 **Polynomials:** Solving Quadratics	Discriminant: ♦ Given the nature of the roots of an equation, use the discriminant to find an unknown	
	Discriminant: ➢ Solve quadratic inequalities, $ax^2 + bx + c \geq 0$ or (≤ 0)	
	Intersection: ➢ Finding the coordinates of the point(s) of intersection of a straight line and a curve or of two curves	As far as possible, solutions of algebraic equations should be linked to a graph of function(s), with learners encouraged to make such connections. (Use of graphic calculators/refer to diagram in question/ sketch diagrams to check solutions.)

3. Topic Skill	Subskill	Support notes (from SQA)
A1.3 **Recurrence Relations:** Modelling situations using sequences	♦ Determining a recurrence relation from given information and using it to calculate a required term ♦ Finding and interpreting the limit of a sequence, where it exists	Where possible, use examples from real-life situations such as where concentrations of chemicals/medicines are important.

4. Topic Skill	Subskill	Support notes (from SQA)
EF1.3 **Functions:** Identifying and sketching related functions	♦ Identify and sketch a function after a transformation of the form $kf(x)$, $f(kx)$, $f(x)+k$, $f(x+k)$ or any combination of these	Use of graphic calculators to explore various transformations. Learners should be able to recognise a function from its graph. Interpret formulae/equations for maximum/minimum values and when they occur.
	➢ Sketch $y = f'(x)$ given the graph of $y = f(x)$	
	➢ Sketch the inverse of a logarithmic or an exponential function	
	➢ Completing the square in a quadratic expression where the coefficient of x^2 is non - unitary	
EF1.3 **Functions:** Determining composite and inverse functions	♦ Determine a composite function given $f(x)$ and $g(x)$, where $f(x)$ and $g(x)$ can be trigonometric, logarithmic, exponential or algebraic functions - including basic knowledge of domain and range	$f(g(x))$ where $f(x)$ is a trigonometric/logarithmic function and $g(x)$ is a polynomial.
	♦ $f^{-1}(x)$ of functions	Learners should be aware that $f(g(x)) = x$ implies that $f(x)$ and $g(x)$ are inverses.
	➢ Know and use the terms domain and range	

5. Topic Skill	Subskill	Support notes (from SQA)
RC1.1 **Polynomials:** Solving algebraic equations	♦ Factorising a cubic polynomial expression with unitary x^3 coefficient	Strategies for factorising polynomials, ie synthetic division, inspection, algebraic long division.
	➢ Factorising a cubic or quartic polynomial expression with a non-unitary coefficient of the highest power	Factorising quadratic at National 5 or in previous learning led to solution(s) which learners can link to graph of function.
	Solving polynomial equations: ♦ Cubic with unitary x^3 coefficient	Identifying when an expression is not a polynomial (negative/fractional powers). Recognise repeated root is also a stationary point.

	Solving polynomial equations: ➢ Cubic or quartic with non-unitary coefficient of the highest power	Emphasise meaning of solving $f(x) = g(x)$. Learners should encounter the Remainder Theorem and how this leads to the fact that for a polynomial equation, $f(x) = 0$, if $(x-h)$ is a factor of $f(x)$, h is a root of the equation and vice versa. Learners' communication should include a statement such as 'since $f(h) = 0$' or 'since remainder is 0'. Learners should also experience divisors/factors of the form $(ax-b)$.

6. Topic Skill	Subskill	Support notes (from SQA)
EF1.1 **Logarithms:** Manipulating algebraic expressions	♦ Simplifying an expression using the laws of logarithms and exponents	Link logarithmic scale to science applications, eg decibel scale for sound, Richter scale of earthquake magnitude, astronomical scale of stellar brightness, acidity and pH in chemistry and biology. Note link between scientific notation and logs to base 10.
	♦ Solving logarithmic and exponential equations	
	♦ Using the laws of logarithms and exponents	
	➢ Solve for a and b equations of the following forms, given two pairs of corresponding values of x and y: $\log y = b\log x + \log a$, $y = ax^b$ and, $\log y = x\log b + \log a$, $y = ab^x$	Real-life contexts involving logarithmic and exponential characteristics, eg rate of growth of bacteria, calculations of money earned at various interest rates over time, decay rates of radioactive materials.
	➢ Using a straight line graph to confirm relationships in the form $y = ax^b$ and $y = ab^x$	
	➢ Model mathematically situations involving the logarithmic or exponential function	

Topic Skill	Subskill	Support notes (from SQA)
EF1.4 **Vectors:** Determining vector connections Working with vectors	♦ Determining the resultant of vector pathways in three dimensions	Learners should work with vectors in both two and three dimensions.
	♦ Working with collinearity	In order to 'show' collinearity, communication should include mention of parallel vectors and 'common point'.
	♦ Determining the coordinates of an internal division point of a line	Distinction made between writing in coordinate and component form.
	♦ Evaluate a scalar product given suitable information and determining the angle between 2 vectors	Also, introduce the zero vector. Perpendicular and distributive properties of vectors should be investigated, eg If $\lvert\mathbf{a}\rvert$, $\lvert\mathbf{b}\rvert \neq 0$ then $\mathbf{a} \cdot \mathbf{b} = 0$ if and only if the directions of a and b are at right angles. Example of broader application: sketch a vector diagram of Course Support Notes for Higher Mathematics Course 17 the three forces on a kite, when stationary: its weight, force from the wind (assume normal to centre of kite inclined facing the breeze) and its tethering string. These must sum to zero.
	➢ Apply the properties of the scalar product	
	➢ Using unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ as a basis	

Topic Skill	Subskill	Support notes (from SQA)
EF1.2 **Trigonometry:** Manipulating trigonometric expressions	Application of: ♦ The addition or double angle formulae	Learners can be shown how formulae for $\cos(\alpha+\beta)$ and $\sin(\alpha+\beta)$ can be used to prove formulae for $\sin 2\alpha$, $\cos 2\alpha$ cos2□ , $\tan(\alpha+\beta)$ Emphasise the distinction between $\sin x^{\circ}$ and $\sin x$ (degrees and radians). Learners should be given practice in applying the standard formulae, eg expand $\sin 3x$ or $\cos 4x$. Learners should be exposed to geometric problems which require the use of addition or double angle formulae. Example of use in science: a train of moving water waves of wavelength λ has a profile $y=\sin\left\{2\pi\left[\frac{t}{T}-\frac{x}{\lambda}\right]\right\}$
	Application of: ♦ Trigonometric identities	
	Convert $a\cos x+b\sin x$ to $k\cos(x\pm\alpha)$ or $k\sin(x\pm\alpha)$, $k>0$ ♦ α in 1st quadrant	
	Convert $a\cos x+b\sin x$ to $k\cos(x\pm\alpha)$ or $k\sin(x\pm\alpha)$, $k>0$ ➢ α in any quadrant	

Topic Skill	Subskill	Support notes (from SQA)
RC1.2 **Trigonometry:** Solving trigonometric expressions	♦ Solve trigonometric equations in degrees, involving trigonometric formulae, in a given interval	Link to trigonometry of Expressions and Functions Unit. Real-life contexts should be used whenever possible. Solution of trigonometric equations could be introduced graphically. Recognise when a solution should be given in radians (eg $0\le x\le\pi$). In the absence of a degree symbol, radians should be used. A possible application is the refraction of a thin light beam passing from air into glass. Its direction of travel is bent towards the line normal to the surface, according to Snell's law.
	➢ Solving trigonometric equations in degrees or radians, including those involving the wave function or trigonometric formulae or identities, in a given interval	

Topic Skill	Subskill	Support notes (from SQA)
A1.2 **Circle:** Applying algebraic skills to circles	♦ Determining and using the equation of a circle	Link to work on discriminant (one point of contact). Develop equation of circle (centre the origin) from Pythagoras, and extend this to circle with centre (a,b) or relate to transformations. Demonstrate application of discriminant. Learners made aware of different ways in which more than one circle can be positioned, eg intersecting at one/two/no points, sharing same centre (concentric), one circle inside another. Practice in applying knowledge of geometric properties of circles in finding related points (eg stepping out method). Solutions should not be obtained from scale drawings.
	♦ Using properties of tangency in the solution of a problem	
	➢ Determining the intersection of circles or a line and a circle	

Topic Skill	Subskill	Support notes (from SQA)
RC1.3, RC1.4, A1.4 **Calculus:** Differentiating functions	♦ Differentiating an algebraic function which is, or can be simplified to, an expression in powers of x	Examples from science using the terms associated with rates of change, eg acceleration, velocity.
	♦ Differentiating $k\sin x$, $k\cos x$	
	➢ Differentiating a composite function using the chain rule	
RC1.3, RC1.4, A1.4 **Calculus:**	♦ Determining the equation of a tangent to a curve at a given point by differentiation	Learners should know that the gradient of a curve at a point is defined to be the gradient of the tangent to the curve at that point.

Using differentiation to investigate the nature and properties of functions	➢ Determine where a function is strictly increasing/decreasing	Learners should know when a function is either strictly increasing, decreasing or has a stationary value, and the conditions for these.
	➢ Sketching the graph of an algebraic function by determining stationary points and their nature as well as intersections with the axes and behaviour of $f(x)$ for large positive and negative values of x	The second derivative or a detailed nature table can be used. Stationary points should include horizontal points of inflexion.

Topic Skill	Subskill	Support notes (from SQA)
RC1.3, RC1.4, A1.4 **Calculus:** Integrating functions	♦ Integrating an algebraic function which is, or can be simplified to, an expression in powers of x	Know the meaning of the terms integral, integrate, constant of integration, definite integral, limits of integration, indefinite integral, area under a curve. Know that if $f(x) = F'(x)$ then $\int_a^b f(x)dx = F(b) - F(a)$ and $\int f(x)dx = F(x) + C$ where C is the constant of integration. Could be introduced by anti-differentiation. Learners should experience integration of $\cos^2 x$ and $\sin^2 x$ using $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
	♦ Integrating functions of the form $f(x) = (x+q)^n$ where $n \neq -1$	
	♦ Integrating functions of the form $f(x) = p\cos x$ and $f(x) = p\sin x$	
	➢ Integrating functions of the form $f(x) = (px+q)^n$ where $n \neq -1$	
	➢ Integrating functions of the form $f(x) = p\cos(qx+r)$ and $f(x) = p\sin(qx+r)$	
	➢ Solving differentiation equations of the form $\frac{dy}{dx} = f(x)$	
RC1.3, RC1.4, A1.4 **Calculus:** Using integration to calculate definite integrals	♦ Calculating definite integrals of polynomial functions with integer limits	Extend to area between the curve between the limits
	➢ Calculating definite integrals of functions with limits that are integers, radians, surds or fractions	

Topic Skill	Subskill	Support notes (from SQA)
RC1.3, RC1.4, A1.4 **Calculus:** Applying differential calculus	♦ Determine the optimal solution for a given problem	Max/min problems applied in context, eg minimum amount of card for creating a box, maximum output from machines. Rate of change linked to science. Optimisation in science, eg an aeroplane cruising at speed v at a steady height, has to use power to push air downwards to counter the force of gravity, and to overcome air resistance to sustain its speed. The energy cost per km of travel is given approximately by: $E = Av^2 + Bv^2$ (A and B depend on the size and weight of the plane). At the optimum speed $\frac{dE}{dv} = 0$, thus get an expression for v_{opt} in terms of A and B.
	➢ Determine the greatest/least values of a function on a closed interval	
	➢ Solving problems using rates of change	

Topic Skill	Subskill	Support notes (from SQA)
RC1.3, RC1.4, A1.4 **Calculus:** Applying integral calculus	♦ Finding the area between a curve and the $x-$axis.	Develop from Relationships and Calculus Unit. Use of graphical calculators for an investigative approach. Area between curves by subtraction of individual areas – use of diagrams, graphing packages. Reducing area to be determined to smaller components in order to estimate segment of area between curve and $x-$
	♦ Finding the area between a straight line and a curve or two curves	

		axis. Use of area formulae (triangle/rectangle) in solving such problems.
	➢ Determine and use a function from a given rate of change and initial conditions	A practical application of the integral of $\frac{1}{x^2}$ is to calculate the energy required to lift an object from the earth's surface into space. The work energy required is $E = \int F dr$ where F is the force due to the earth's gravity and r is the distance from the centre of the earth. For a 1 kg object $E = -\int \left(\frac{GM}{r^2} \right) dr$ where M is the mass of the earth and G is the universal gravitational constant. $GM = 4 \cdot 0 \times 10^{14}\,\mathrm{m^3 s^{-2}}$. The integration extends from $r = 6 \cdot 4 \times 10^6$ m (the radius of the earth) to infinity.