

Higher Applications Revision

Solutions

① A(3,0) B(5,2)

(a) Perpendicular bisector of AB

$$M_{AB} = \frac{2-0}{5-3} = \frac{2}{2} = 1 \quad \text{mid} = -1 \quad \text{Mid}_{AB} \Rightarrow \left(\frac{3+5}{2}, \frac{0+2}{2} \right) = (4,1)$$

$$y - b = m(x - a)$$

$$y - 1 = -1(x - 4)$$

$$y - 1 = -x + 4$$

$$\underline{y + x - 5 = 0}$$

(b) $y + 2x = 6$ = median from A

T° Pt. of Intersection

$$\begin{array}{l} y + x = 5 \\ \hline y + 2x = 6 \end{array}$$

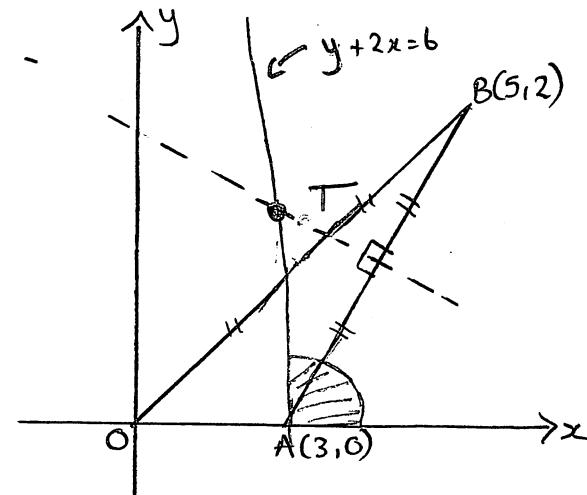
$$-x = -1$$

$$\underline{x = 1}$$

$$y + 1 = 5$$

$$\underline{y = 4}$$

$$T = \underline{(1,4)}$$



(c) $M = \tan \theta$

$$\text{AT } \theta \quad y + 2x = 6$$

$$y = -2x + 6$$

$$m = -2$$

$$-2 = \tan \theta \quad \text{Obtuse angle}$$

$$\tan^{-1}(2) = 63.4^\circ$$

$$180^\circ - 63.4^\circ = \underline{116.6^\circ}$$

Q2. (a) L₁ ° $3x - 4y + 2 = 0 \quad \left(\frac{a}{b}\right)$

$$-4y = -3x - 2$$

$$y = \frac{3}{4}x + \frac{1}{2} \quad \underline{m = \frac{3}{4}}$$

$$y - b = m(x - a)$$

$$y - 2 = \frac{3}{4}(x - (-1)) \quad \times 4$$

$$4y - 8 = 3x + 3$$

$$\underline{3x - 4y + 11 = 0}$$

|| lines \Rightarrow equal gradients

(b) PQ ⊥ L₂

$$L_2: 2x - 4y = 10 \quad \left(\frac{a}{b}\right)$$

$$-4y = -2x + 10$$

$$y = \frac{1}{4}x - \frac{10}{4}$$

$$\underline{m = \frac{1}{4}}$$

$$\perp m = -4$$

$$y - 2 = -4(x - 0)$$

$$y - 2 = -4x$$

$$\underline{y + 4x - 2 = 0}$$

(c) L₁ ° $3x - 4y + 2 = 0$

L₂ ° $x - 4y = 10$

Pt. of
Intersection
 $\underline{\underline{= (-6, -4)}}$

$$\begin{array}{l} 3x - 4y = -2 \\ x - 4y = 10 \\ \hline 2x = -12 \\ x = -6 \end{array}$$

$$3(-6) - 4y = -2$$

$$-18 - 4y = -2$$

$$-4y = 16$$

$$y = -4$$

$$Q3. M = \tan \theta$$

$$M = \tan 150^\circ$$

$$\tan 150^\circ = -0.58 = m = \left(-\frac{1}{\sqrt{3}}\right)$$

Q4. (a) Equation of QR

$$Q(5, 6) \quad P(7, 2)$$

PQRS = Rectangle

QR \perp QP

$$M_{QP} = \frac{2-6}{7-5} = \frac{-4}{2} = -2$$

$$\perp m = \frac{1}{2} \quad \text{Use pt. Q}$$

$$y - b = m(x - a)$$

$$y - 6 = \frac{1}{2}(x - 5)$$

$$\times 2$$

$$2y - 12 = x - 5$$

$$2y - x - 7 = 0$$

Q5. ABCD = Rhombus

$$AD \parallel y = 5$$

$$(i) BC \perp AD \quad B = (2, 8)$$

$$y = 5 \Rightarrow m = 0$$

$\perp m = \text{undefined}$

$$\Rightarrow \text{Equation } \hat{=} \underline{x = 2}$$

$$(ii) \text{ Pt. of Intersection} = \underline{(2, 4)}$$

$$Q7. x - 2y = 3 \quad 3y + 6x = 7$$

$$\text{Perpendicular} \Rightarrow M_1 \times M_2 = -1$$

$$x - 2y = 3$$

$$-2y = -x + 3$$

$$y = \frac{1}{2}x - \frac{3}{2}$$

$$M_1 = \frac{1}{2}$$

$$3y = -6x + 7$$

$$y = -2x + \frac{7}{3}$$

$$y = -2x + \frac{7}{3}$$

$$M_2 = -2$$

$$\frac{1}{2} \times -2 = -\frac{2}{2} = -1 \Rightarrow \text{Lines are perpendicular.}$$

(b) $x + 3y = 13$ intersects QR at T
Find T

$$\begin{array}{r} x + 3y = 13 \\ -x + 2y = 7 \\ \hline 5y = 20 \\ y = 4 \end{array} \quad \begin{array}{l} x + 3(4) = 13 \\ x + 12 = 13 \\ x = 1 \end{array}$$

$$T \underline{(1, 4)}$$

$$Q6. (a) 3x + 2y = 5$$

$$2y = -3x + 5$$

$$y = -\frac{3}{2}x + \frac{5}{2}$$

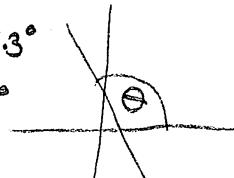
$$M = \underline{-\frac{3}{2}}$$

$$(b) M = \tan \theta$$

$$\tan^{-1}\left(\frac{3}{2}\right) = 56.3^\circ$$

$$\theta = 180^\circ - 56.3^\circ$$

$$\theta = \underline{123.7^\circ}$$



$$\textcircled{8} \text{ (i) } E = \begin{pmatrix} a & b \\ 7 & 7 \end{pmatrix}$$

$$\text{h.o } x - y = 2 \\ y = x - 2 \\ m = 1 \quad +m = -1$$

$$y - b = m(x - a) \\ y - 7 = 1(x - 7) \\ y - 7 = -x + 7 \\ \text{EG: } y + x - 14 = 0$$

(ii) Pt. of Intersection

$$\begin{array}{l} x - y = 2 \\ + x + y = 14 \\ \hline 2x = 16 \\ x = 8 \end{array} \quad \begin{array}{l} 8 - y = 2 \\ -y = -6 \\ y = 6 \end{array}$$

K(8, 6)

$$\textcircled{9} \text{ (a) Mid}_{AB} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(3\frac{1}{2}, 2\frac{1}{2} \right)$$

$$\left(\frac{4 + x_2}{2}, \frac{4 + y_2}{2} \right) = \left(3\frac{1}{2}, 2\frac{1}{2} \right)$$

$$\frac{4 + x_2}{2} = 3\frac{1}{2} \\ 4 + x_2 = 7 \\ x_2 = 3$$

A(3, 1)

$$\frac{4 + y_2}{2} = 2\frac{1}{2} \\ 4 + y_2 = 5 \\ y_2 = 1$$

$$\text{(b) } M = \tan \theta \quad \begin{matrix} x_1 & y_1 \\ 3 & 4 \\ A(3, 1) \end{matrix} \quad \begin{matrix} x_2 & y_2 \\ 7 & 2 \\ C(7, 2) \end{matrix}$$

$$M = \frac{2-4}{7-3} = \frac{1}{4}$$

$$\begin{aligned} m &= \tan \theta \\ \frac{1}{4} &= \tan \theta \\ \theta &= \tan^{-1}(\frac{1}{4}) \\ \underline{\theta = 14^\circ} \end{aligned}$$

$$Q10. BC \circledast y = x + 4$$

$$(i) AD \quad A(3, 9) \quad AD \perp BC$$

$$y = x + 4 \\ m=1 \quad \perp m=-1$$

$$y - b = m(x - a) \\ y - 9 = 1(x - 3) \\ y - 9 = x - 3$$

$$AD \circledast y = -x + 12$$

$$(ii) \quad \begin{array}{r} y + 9c = 12 \\ y - 9c = 4 \\ \hline 2y = 16 \\ y = 8 \end{array}$$

$$8 + 2c = 12 \\ 2c = 4$$

(4, 8)

1.2

$$\textcircled{1} \quad (a) \quad 2x - y + 5 = 0 \quad \textcircled{2} \quad x^2 + y^2 - 6x - 2y - 30 = 0$$

Find P and Q

$$2x - y + 5 = 0$$

$$\textcircled{1} \quad y = 2x + 5$$

sub. \textcircled{1} into \textcircled{2}

$$x^2 + (2x+5)^2 - 6x - 2(2x+5) - 30 = 0$$

$$x^2 + 4x^2 + 20x + 25 - 6x - 4x - 10 - 30 = 0$$

$$5x^2 + 10x - 15 = 0$$

$$x^2 + 2x - 3 = 0$$

$$(x - 1)(x + 3) = 0$$

$$x = 1 \quad x = -3$$

$$\begin{array}{ll} y = 2(1) + 5 & y = 2(-3) + 5 \\ y = 7 & y = -1 \end{array}$$

$$Q(1, 7) \quad P(-3, -1)$$

$$(b) \quad x^2 + y^2 - 6x - 2y - 30 = 0$$

$$\text{centre} = (3, 1)$$

$$\text{radius} = \sqrt{3^2 + 1^2 + 30} = \sqrt{40} = 2\sqrt{10}$$

$$\text{Mid}_{PQ} = \left(\frac{1 + (-3)}{2}, \frac{7 + (-1)}{2} \right) = (-1, 3)$$

$$(3, 1) \xrightarrow{-4 \quad +2} (-1, 3) \xrightarrow{-4 \quad +2} (-5, 5)$$

$$\text{radius} = 2\sqrt{10} \quad (\text{congruent})$$

$$(x+5)^2 + (y-5)^2 = (2\sqrt{10})^2$$

$$(x+5)^2 + (y-5)^2 = 400$$

Q2. (a) (i) $y = 3 - x$
 $x^2 + y^2 + 14x + 4y - 19 = 0$
 $x^2 + (3-x)^2 + 14x + 4(3-x) - 19 = 0$
 $x^2 + 9 - 6x + x^2 + 14x + 12 - 4x - 19 = 0$
 $2x^2 + 4x + 2 = 0$
 $x^2 + 2x + 1 = 0$
 $(x+1)(x+1) = 0$

$\underline{x = -1}$ only 1 root \Rightarrow tangent

$y = 3 - (-1) = 4$
(ii) $P \underline{(-1, 4)}$

Q3. $x^2 + y^2 + 6x + 4y - 12 = 0$

A \therefore Centre = $(-3, -2)$
radius = $\sqrt{3^2 + 2^2 + 12} = \sqrt{25} = 5$
 $x^2 + y^2 - 6x - 12y + 20 = 0$
B \therefore Centre = $(3, 6)$
radius = $\sqrt{3^2 + 6^2 - 20} = \sqrt{25} = 5$

(a) $P = \text{midpoint } AB$
 $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right) = \left(\frac{-3+3}{2}, \frac{-2+6}{2} \right) = \underline{(0, 2)}$

(b) $|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
 $= \sqrt{(3 - (-3))^2 + (6 - (-2))^2}$
 $= \sqrt{36 + 64} = \sqrt{100} = \underline{10}$

Q4. C $(-2, 3)$ P $(1, b)$
(a) $(x-a)^2 + (y-b)^2 = r^2$
 $(1 - (-2))^2 + (b - 3)^2 = r^2$
 $= 3^2 + 3^2 = r^2$
 $18 = r^2$
 $r = \sqrt{18} = 3\sqrt{2}$

$\underline{(x+2)^2 + (y-3)^2 = 18}$

(b) $P(1, b) \rightarrow C(-2, 3) \rightarrow Q \underline{(-5, 0)}$
 $M_{CQ} = \frac{0 - 3}{-5 - (-2)} = \frac{-3}{-3} = 1 \quad \perp m = -1$
 $y - b = m(x - a)$
 $y - 0 = -1(x + 5)$
 $\underline{y = -x - 5}$

Q5. (i) Equation of C₁

$r=2$ centre = (2, 2)

$$(x-2)^2 + (y-2)^2 = 4$$

(ii) $r_{C_1} = r_{C_2}$

centre = (3, 3) $r=2$

$$\underline{(x-3)^2 + (y-3)^2 = 4}$$

1.3

Q1. $U_{n+1} = 0.4U_n - 5$ $U_0 = 8$

$$U_1 = 0.4(8) - 5 = -1.8$$

$$U_2 = 0.4(-1.8) - 5 = -5.72$$

$$U_3 = 0.4(-5.72) - 5 = \underline{-7.288}$$

Q2. $U_1 = 4$ $U_2 = 7$ $U_3 = 16$

$$U_{n+1} = mU_n + c$$

$$7 = 4m + c$$

$$16 = 7m + c$$

$$\underline{-9 = -3m}$$

$$7 = 4(3) + c$$

$$\underline{m = 3}$$

$$\underline{-5 = c}$$

$$U_{n+1} = 3U_n - 5$$

Q3. $U_{n+1} = 0.8U_n + 0.5$

(a) $L = \frac{b}{1-a} = \frac{0.5}{1-0.8} = \frac{0.5}{0.2} = \underline{2\frac{1}{2} \text{ metres}}$

(b) $\frac{b}{1-a} = 2$

$$\frac{0.5}{1-a} = 2$$

$$\times (1-a)$$

$$x(1-a)$$

$$0.5 = 2 - 2a$$

$$-1.5 = -2a$$

$$\underline{a = 0.75}$$

Trim trees by 25 %

$$Q4. P_{n+1} = aU_n + 12$$

$$Q_{n+1} = 0.5U_n + 4$$

some limit at $n \rightarrow \infty$, find a

$$L = \frac{b}{1-a}$$

$$\Rightarrow L = \frac{4}{1-0.5} = \frac{4}{0.5} = 8$$

$$L = \frac{12}{1-a} = \frac{8}{x(1-a)}$$

$$12 = 8 - 8a$$

$$4 = -8a$$

$$a = \underline{-0.5}$$

$$Q6. Killpest \% U_{n+1} = 0.35U_n + 500$$

$$pestkill \% U_{n+1} = 0.15U_n + 650$$

$$L_1 = \frac{500}{1-0.35} = 769.23$$

$$L_2 = \frac{650}{1-0.15} = 764.71$$

Pestkill will be more effective because
 $\underline{769.23} > \underline{764.71}$

$$Q5. U_{n+1} = 0.3U_n + 35$$

$$L = \frac{b}{1-a} = \frac{35}{0.7} = \underline{50g}$$

Yes as the maximum level the drug reaches will be 50g < 54g.

$$Q7. (d) L = \frac{b}{1-a}$$

$-1 < L < 0.522$ \leftarrow
 \Rightarrow a limit exists

$$L = \frac{25}{1-0.522}$$

$$L = 52.3g < 55g$$

\Rightarrow no maximum length of time
 Poor 004 \therefore

$$(e) U_{n+1} = (0.85)(0.85)(0.85) \\ (0.85)U_n + 25$$

$$U_{n+1} = (0.85)^4 U_n + 25$$

$$\underline{U_{n+1} = 0.522U_n + 25}$$

He will always be above 2g after 3 doses.
 It really depends on how long they want to interrogate for!

$$(a) 1hr = 0.85(25) = 21.25g \quad 1^{st} \text{ dose}$$

$$2hrs = 0.85(21.25) = 18.0625$$

$$3hrs = 0.85(18.0625) = 15.35$$

$$4hrs = 0.85(15.35) = 13.05g$$

$$(b) 13.05 + 25 = 38.05g \quad \text{after } 2^{nd} \text{ dose}$$

$$5hrs 0.85(38.05) = 32.34g$$

$$6hrs 0.85(32.34) = 27.49$$

$$7hrs 0.85(27.49) = 23.3665$$

$$8hrs 0.85(23.3665) + 25 = 44.86 \quad \text{after } 3^{rd} \text{ dose}$$

$$19.86 < 20$$

$$\text{after 11 hours } (0.85)^3 \times 44.86 = 38.13$$

$$0.85(38.13) = 32.41$$

$$③ K_{n+1} = aK_n + b$$

$$U_1 = 6 \quad U_2 = 12 \quad U_3 = 21$$

$$(i) \quad 12 = 6a + b$$

$$\textcircled{O} \quad 21 = 12a + b$$

$$\underline{-9 = -6a}$$

$$\underline{a = \frac{2}{3}}$$

$$12 = 6\left(\frac{2}{3}\right) + b$$

$$12 = 4 + b$$

$$\underline{b = 8}$$

$$(ii) \quad L = \frac{b}{1-a} = \frac{8}{1-\frac{2}{3}} = \underline{\underline{24}}$$

1.4

$$Q1. \quad y = 2x + 3 \quad y = x^3 + 3x^2 + 2x + 3$$

$$\begin{array}{r} 2x + 3 = x^3 + 3x^2 + 2x + 3 \\ -2x \quad -3 \\ \hline 0 = x^3 + 3x^2 \end{array}$$

$$-2x \quad -3$$

$$0 = x^2(x + 3)$$

$$x = 0 \quad x = -3$$

$$y = 3 \quad y = 2(-3) + 3$$

$$A(0, 3) \quad B(-3, -3)$$

Area = \int Upper - Lower

$$\Rightarrow \int_{-3}^0 [x^3 + 3x^2 + 2x + 3] - (2x + 3) dx$$

$$= \int_{-3}^0 (x^3 + 3x^2) dx$$

$$= \left[\frac{x^4}{4} + \frac{3x^3}{3} \right]_{-3}^0 = \left[\frac{0^4}{4} + 0^3 \right] - \left[\frac{(-3)^4}{4} + (-3)^3 \right]$$

$$= [0] - \left[\frac{81}{4} - 27 \right]$$

$$- \left[-\frac{27}{4} \right] = \underline{\underline{\frac{27}{4} \text{ units}^2}}$$

Q2. (a) S.A = 12 units²

$$S.A = 2xh + 2xh + xh + xh + 2x^2$$

$$S.A = 6xh + 2x^2$$

$$12 = 6xh + 2x^2$$

$$12 - 2x^2 = 6xh$$

$$\frac{12 - 2x^2}{6x} = h$$

$$\frac{6 - x^2}{3x} = h$$

$$V = L \times B \times H$$

$$V = 2x^2 \times \left(\frac{6 - x^2}{3x} \right)$$

$$V = \frac{12x - 2x^3}{3} = 2x \left(\frac{6 - x^2}{3} \right) = \frac{2}{3}x(6 - x^2)$$

(b) Value for x for which volume = maximum

$$0 < x < 12$$

End-points

$$V(0) = \frac{2}{3}(0)(6 - 0^2) = 0$$

$$V(12) = \frac{2}{3}(12)(6 - 12^2) = -1104 \quad \cancel{V(0)}$$

$$V(x) = \frac{2}{3}x(6 - x^2) = 4x - \frac{2}{3}x^3$$

$$V'(x) = 4 - 2x^2 = 0$$

$$4 = 2x^2$$

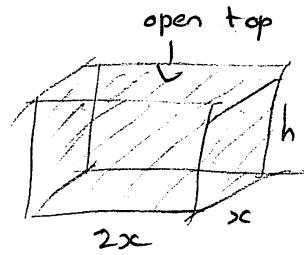
$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

x	(-1.5)	(-1)	(0)	(1)	(1.5)
$\frac{dy}{dx}$	-	0	+	+	0
Slope	\	-	/	/	\

\Rightarrow Maximum at $x = \underline{\sqrt{2}}$

$$\text{Volume} = \frac{2}{3}(\sqrt{2})(6 - (\sqrt{2})^2) = 3.77 \text{ units}^3$$



Q3. Area = \int Upper - Lower

$$A = \int_0^5 [(1+10x-2x^2) - (1+5x-x^2)] dx$$

$$A = \int_0^5 (5x - x^2) dx$$

Limits :

$$1+10x-2x^2 = 1+5x-x^2$$

$$x^2 - 5x = 0$$

$$x(x-5) = 0$$

$$\underline{x=0} \quad \underline{x=5}$$

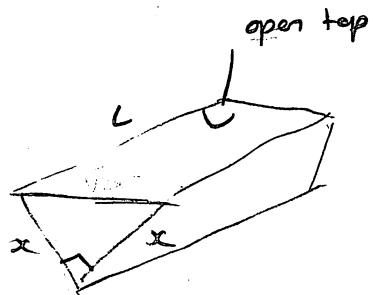
$$= \left[\frac{5x^2}{2} - \frac{x^3}{3} \right]_0^5 = \left[\frac{5(5)^2}{2} - \frac{5^3}{3} \right] - \left[\frac{5(0)^2}{2} - \frac{0^3}{3} \right]$$

$$= \frac{125}{2} - \frac{125}{3} = \underline{\underline{\frac{125}{6}}} \text{ units}^2$$

Q4. Capacity = 108 Litres = 108000 mL = 108000 cm³

(a) Show that S.A is

$$A(x) = x^2 + \underline{\underline{\frac{432000}{x}}}$$



$$V = \frac{1}{2} \times L \times B \times H$$

$$S.A = \frac{1}{2}x^2 + \frac{1}{2}x^2 + xL + xL$$

$$V = \frac{1}{2} \times L \times x \times x$$

$$S.A = x^2 + 2xL$$

$$V = \frac{1}{2} L x^2$$

$$S.A = x^2 + 2x \left(\frac{216000}{x^2} \right)$$

$$108000 = \frac{1}{2} L x^2$$

$$L = \underline{\underline{\frac{216000}{x^2}}}$$

$$S.A = x^2 + \underline{\underline{\frac{432000}{x}}}$$

$$(b) A(x) = x^2 + 432000x^{-1}$$

$$A'(x) = 2x - \underline{\underline{\frac{432000}{x^2}}} = 0$$

$$2x^3 - 432000 = 0$$

$$2x^3 = 432000$$

$$x^3 = 216000$$

$$x = \sqrt[3]{216000}$$

$$\underline{\underline{x=60}}$$

x	60	60	60+
$\frac{dy}{dx}$	-	0	+
Slope	\	-	/

minimum at $\underline{\underline{x=60}}$

$$Q5. \quad f(x) = x^2 + 2x \quad g(x) = x^3 - x^2 - 6x$$

$$A(4, 24) \quad O(0, 0)$$

Upper - Lower

$$\Rightarrow \int_0^4 [(x^2 + 2x) - (x^3 - x^2 - 6x)] dx$$

$$= \int_0^4 (2x^2 - x^3 + 8x) dx$$

$$= \left[\frac{2x^3}{3} - \frac{x^4}{4} + \frac{8x^2}{2} \right]_0^4 = \left[\frac{2x^3}{3} - \frac{x^4}{4} + 4x^2 \right]_0^4$$

$$\Rightarrow \left[\frac{2(4)^3}{3} - \frac{4^4}{4} + 4(4^2) \right] - [0 - 0 + 0]$$

$$\Rightarrow \frac{128}{3} - 64 + 64 = \underline{\underline{\frac{128}{3} \text{ units}^2}}$$

$$Q6. \quad V = 400 \text{ cm}^3$$

$$(a) \text{ Show } A(r) = 3\pi r^2 + \frac{800}{r}$$

$$V = \pi r^2 h$$

$$400 = \pi r^2 h$$

$$h = \frac{400}{\pi r^2}$$

$$S.A = 2\pi r^2 + \pi r^2 + 2\pi rh$$

$$S.A = 3\pi r^2 + 2\pi r \left(\frac{400}{\pi r^2} \right)$$

$$S.A = 3\pi r^2 + \frac{800}{r}$$

(b) Minimum S.A

$$A(r) = 3\pi r^2 + 800r^{-1}$$

$$A'(r) = 6\pi r - \frac{800}{r^2} = 0$$

$$6\pi r^3 - 800 = 0$$

$$6\pi r^3 = 800$$

$$r^3 = \frac{800}{6\pi}$$

$$r = \sqrt[3]{\frac{800}{6\pi}} \approx 3.5$$

r	3	3.5	3.5	3.5+
$\frac{dy}{dr}$	+	0	-	+
Slope	/	-	/	/

\Rightarrow minimum at $r = \underline{\underline{3.5}}$

Ex. 7.

Q7. $y = x^3 - x^2 - 6x - 2$ $A(1, -8)$

(a) Equation of tangent at A

$$\frac{dy}{dx} = 3x^2 - 2x - 6 \quad x = 1$$

$$\Rightarrow 3(1)^2 - 2(1) - 6 = m$$

$$\underline{\underline{-5 = m}}$$

$$-5x - 3 = x^3 - x^2 - 6x - 2$$

$$x^3 - x^2 - x + 1 = 0$$

$(x-1)$ is a factor

$$y - b = m(x - a)$$

$$y + 8 = -5(x - 1)$$

$$\underline{\underline{y = -5x - 3}}$$

$$\begin{array}{r|rrrr} 1 & 1 & -1 & -1 & 1 \\ & & 1 & 0 & -1 \\ \hline & 1 & 0 & -1 & 0 \end{array}$$

$$x^2 = 1$$

$$(x-1)(x+1)(x-1) = 0$$

$$x = -1 \text{ and } x = 1$$

$$y = -5(-1) - 3$$

$$(1, -8)$$

$$y = 2$$

$$B(\underline{\underline{-1, 2}})$$

(b) Shaded area = $\int_{-1}^1 (\text{Upper} - \text{Lower}) dx$

$$\int_{-1}^1 [(x^3 - x^2 - 6x - 2) - (-5x - 3)] dx$$

$$\int_{-1}^1 (x^3 - x^2 - x + 1) dx$$

$$= \left[\frac{x^4}{4} - \frac{x^3}{3} - \frac{x^2}{2} + x \right]_{-1}^1$$

$$= \left[\frac{1}{4} - \frac{1}{3} - \frac{1}{2} + 1 \right] - \left[\frac{(-1)^4}{4} - \frac{(-1)^3}{3} - \frac{(-1)^2}{2} + (-1) \right]$$

$$= \left[\frac{5}{12} \right] - \left[-\frac{11}{12} \right] = \frac{16}{12} = \frac{4}{3} \text{ units}^2$$

$$Q8. \quad y = x^2(x-5)$$

Roots of $y=0$

$$\begin{aligned} x^2(x-5) &= 0 \\ x^2 &= 0 \quad x = 5 \\ x &\geq 0 \end{aligned}$$

$$\int_0^5 (x^3 - 5x^2) dx$$

$$= \left[\frac{x^4}{4} - \frac{5x^3}{3} \right]_0^5$$

$$= \left[\frac{5^4}{4} - \frac{5(5)^3}{3} \right] - [0] = \frac{625}{4} - \frac{625}{3}$$

$$= -\frac{625}{12} \Rightarrow \text{Area} = \underline{\underline{\frac{625}{12} \text{ units}^2}}$$

$$Q9. \quad y = -x^2 + b \quad y = x^2 - 2$$

Points of intersection of

$$-x^2 + b = x^2 - 2$$

$$-2x^2 = -8$$

$$x^2 = 4$$

$x = \pm 2$ or use graph!

$$(i) \quad \int_{-2}^2 [(-x^2 + b) - (x^2 - 2)] dx$$

$$= \int_{-2}^2 (-2x^2 + 8) dx$$

$$(ii) \quad \int_{-2}^2 (-2x^2 + 8) dx$$

$$= \left[-\frac{2x^3}{3} + 8x \right]_{-2}^2$$

$$\Rightarrow \left[-2\frac{(2)^3}{3} + 8(2) \right] - \left[-2\frac{(-2)^3}{3} + 8(-2) \right]$$

$$\Rightarrow \left[-\frac{16}{3} + 16 \right] - \left[\frac{16}{3} - 16 \right]$$

$$\Rightarrow -\frac{32}{3} + 32 = \underline{\underline{\frac{64}{3} \text{ units}^2}}$$