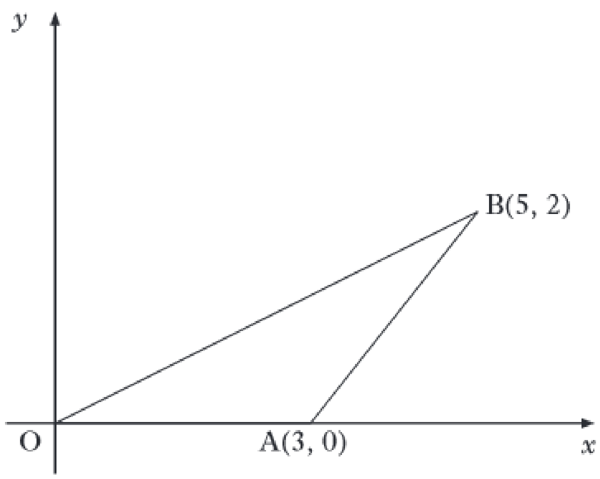
All the questions below contain elements of 2.1 and 2.2

Outcome 1.1: Applying algebraic skills to rectilinear shapes (Straight line)

1.
A(3, 0), B(5, 2) and the origin are the vertices of a triangle as shown in the diagram.



From: SQA

(*a*) Obtain the equation of the perpendicular bisector of AB. 4

(*b*) The median from A has equation $y + 2x = 6$.
Find T, the point of intersection of this median and the perpendicular bisector of AB. 2

(*c*) Calculate the angle that AT makes with the positive direction of the x-axis. 2

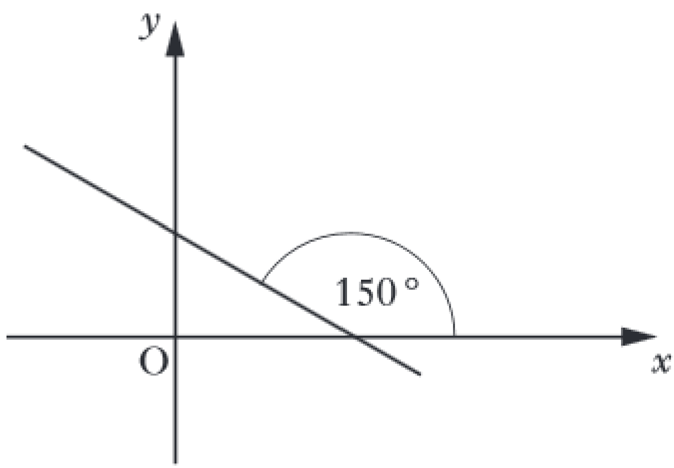
2. (a) State the equation of the line AB which is parallel to L_1: $3x - 4y + 2 = 0$ and passes through the point (-1, 2)

(b) State the equation of the line PQ, which is perpendicular to the line

L_2: $x - 4y = 10$ and passes through (0, 2)

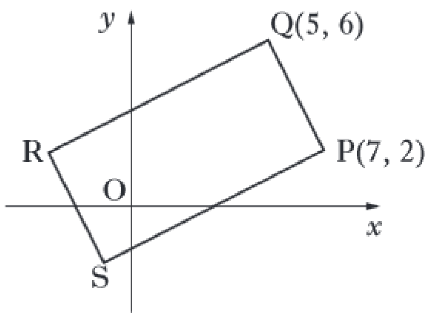
(c) State the point of intersection of the lines L_1 and L_2

3. What is the gradient of the line shown in the diagram?



From: SQA

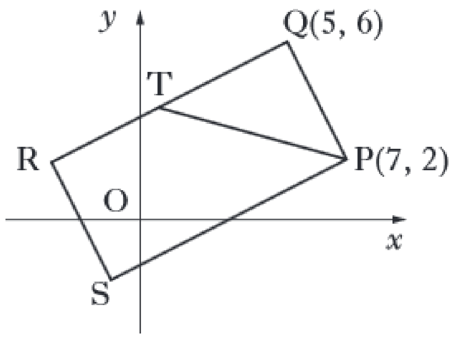
4. The diagram shows rectangle PQRS with P(7, 2) and Q(5, 6).



From: SQA

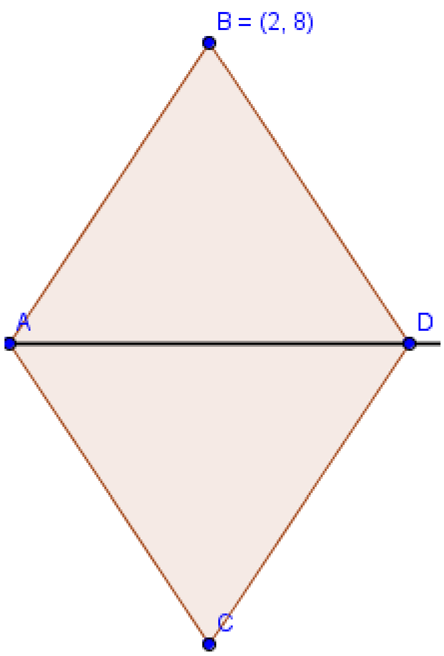
(a) Find the equation of QR. **3**

(b) The line from P with the equation $x + 3y = 13$ intersects QR at T.



Find the coordinates of T. **3**

5. Given the rhombus ABCD, the line AD is $y = 5$.
   (i) State the equation of the line BC.
   (ii) State the point of intersection of the lines AD and BC.



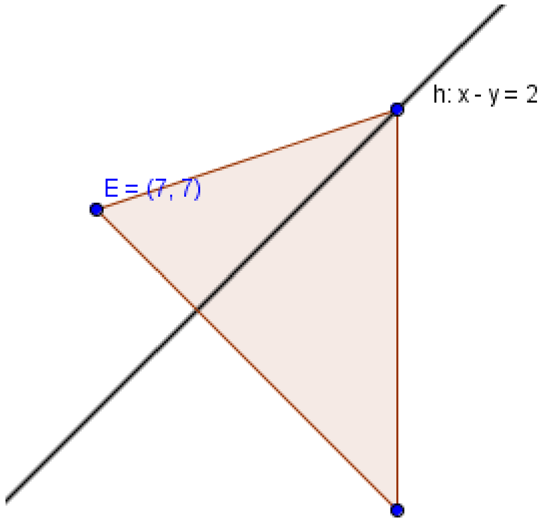
6. (a) State the gradient of the line $3x + 2y = 5$.
   (b) Calculate the size of the angle that the above line makes with the positive direction of the x-axis.

7. Prove that the lines $x - 2y = 3$ and $3y + 6x = 7$ are perpendicular.

8. The line h is the altitude from F through the line EG.

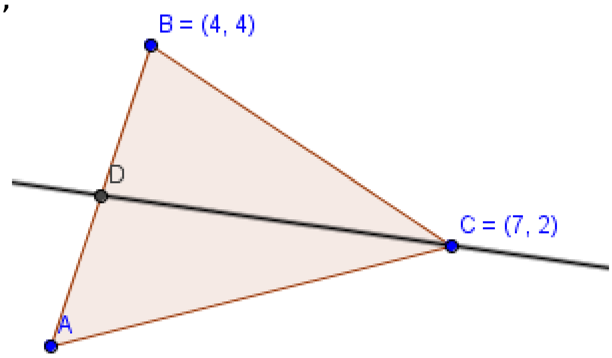
(i) Given that E = (7, 7), state the Equation of the line EG.

(ii) State the point of intersection K of h and EG.



9. (a) Given that CD is the median from C, and the point D is (3½, 2½), find the coordinates of A.

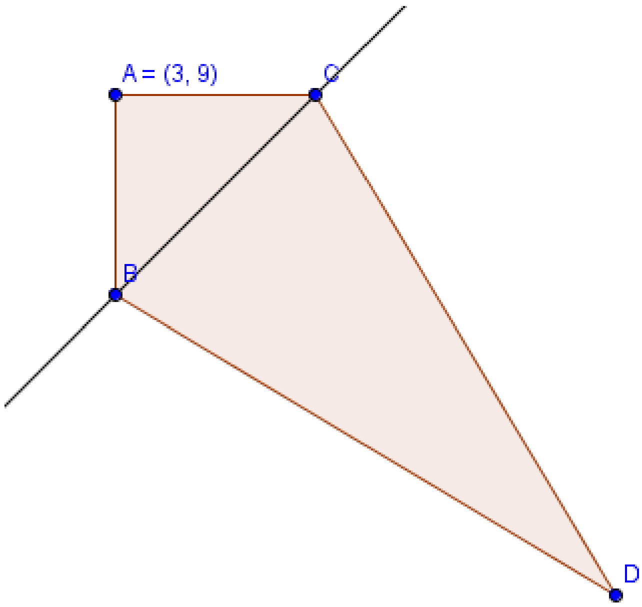
(b) Hence calculate the acute angle that AC makes with the x – axis.



10. Given the Kite ABCD, where the equation of BC is y = x + 4.

(i) State the equation of AD.

(ii) State the point of Intersection of the lines AD and BC.



Outcome 1.2: Applying algebraic skills to circles

1. (*a*) Relative to a suitable set of coordinate axes, Diagram 1 shows the line $2x - y + 5 = 0$ intersecting the circle $x^2 + y^2 - 6x - 2y - 30 = 0$ at the points P and Q.

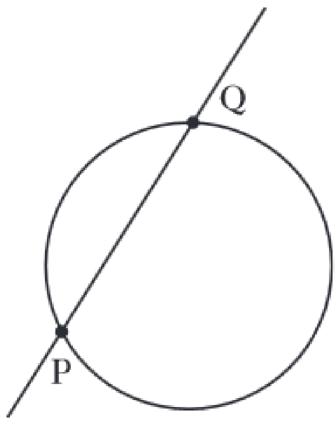
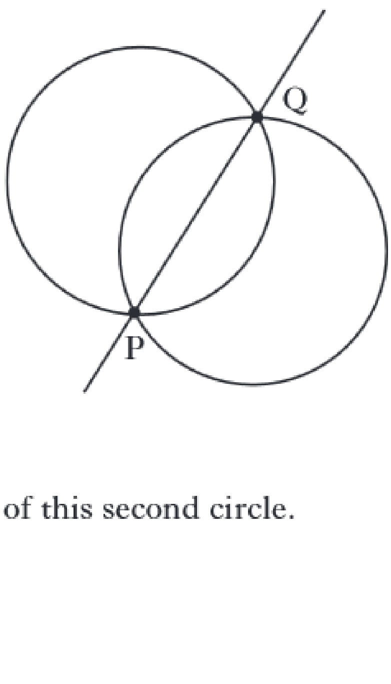


Diagram 1

Find the coordinates of P and Q.

(*b*) Diagram 2 shows the circle from (*a*) and a second congruent circle, which also passes through P and Q.



From: SQA

Diagram 2

Determine the equation of this second circle.

2. (*a*) (i) Show that the line with equation $y = 3 - x$ is a tangent to the circle with equation $x^2 + y^2 + 14x + 4y - 19 = 0$.

(ii) Find the coordinates of the point of contact, P.

From: SQA

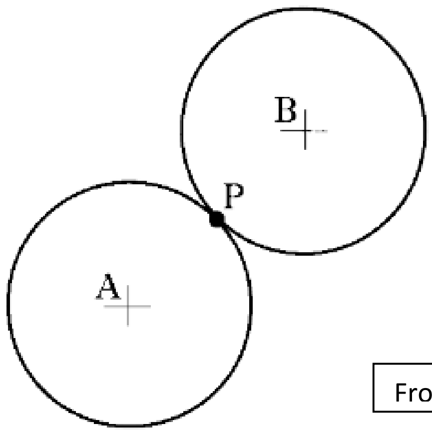
3. Two congruent circles, with centres A and B, touch at P.

Relative to suitable axes, their equations are

$x^2 + y^2 + 6x + 4y - 12 = 0$ and
$x^2 + y^2 - 6x - 12y + 20 = 0.$

(*a*) Find the coordinates of P.

(*b*) Find the length of AB.



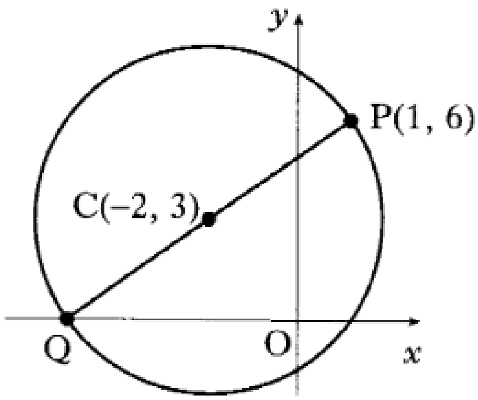
From: SQA

4

A circle has centre C(–2, 3) and passes through P(1, 6).

(*a*) Find the equation of the circle.

(*b*) PQ is a diameter of the circle. Find the equation of the tangent to this circle at Q.

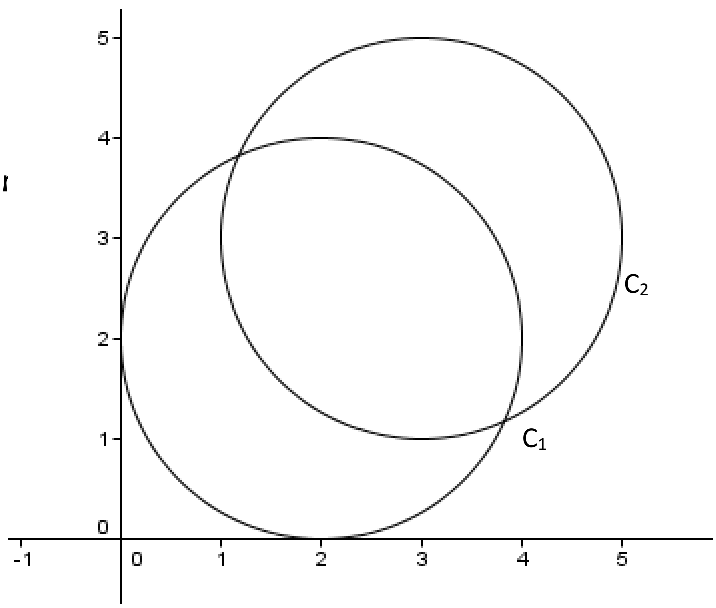


From: SQA

5. The diagram shows two congruent circles. C_1 touches both the x and y axes.

(i) State the equation of the lower circle C_1.

(ii) Given that the centre of C_2 is (3, 3), state the equation of C_2.



Outcome 1.3: Applying algebraic skills to sequences

1. Given the sequence $U_{n+1} = 0.4U_n - 5$, where $U_0 = 8$. State the value of U_2 and U_3.

2. The first three terms of a sequence are 4, 7 and 16.

   The sequence is generated by the recurrence relation

$$u_{n+1} = mu_n + c, \text{ with } u_1 = 4.$$

   Find the values of m and c.

From: SQA

3. A man decides to plant a number of fast-growing trees as a boundary between his property and the property of his next door neighbour. He has been warned, however, by the local garden centre that, during any year, the trees are expected to increase in height by 0·5 metres. In response to this warning he decides to trim 20% off the height of the trees at the start of any year.

   (*a*) If he adopts the "20% pruning policy", to what height will he expect the trees to grow in the long run?

   (*b*) His neighbour is concerned that the trees are growing at an alarming rate and wants assurances that the trees will grow no taller than 2 metres. What is the minimum percentage that the trees will need to be trimmed each year so as to meet this condition?

From: SQA

4. Given $P_{n+1} = aU_n + 12$ and $Q_{n+1} = 0.5U_n + 4$ have the same limit as $n \to \infty$. Find the value of a.

5. A patient is put on medication which needs to be taken once daily. The dose is 35mg. At the end of each day, only 30% of the drug is left in the patient's system. If the level of the drug in the patient's system reaches 54mg, then the patient could die. Is it safe for the patient to continue taking the drug?

6. Trees are sprayed weekly with the pesticide, KILLPEST, whose manufacturers claim it will destroy 65% of all pests. Between the weekly sprayings it is estimated that 500 new pests invade the trees.
A new pesticide, PESTKILL, comes onto the market. The manufacturers claim that it will destroy 85% of existing pests but it is estimated that 650 new pests per week will invade the trees.
Which pesticide will be more effective in the long term ?

From: SQA

7. Secret Agent 004 has been captured and his captors are giving him a 25 milligram dose of a truth serum every 4 hours. 15% of the truth serum present in his body is lost every hour.

(*a*) Calculate how many milligrams of serum remain in his body after 4 hours (that is immediately **before** the second dose is given).

(*b*) It is known that the level of serum in the body has to be continuously above 20 milligrams before the victim starts to confess. Find how many doses are needed before the captors should begin their interrogation.

(*c*) Let u_n be the amount of serum (in milligrams) in his body just **after** his n^{th} dose. Show that $u_{n+1} = 0\cdot522u_n + 25$.

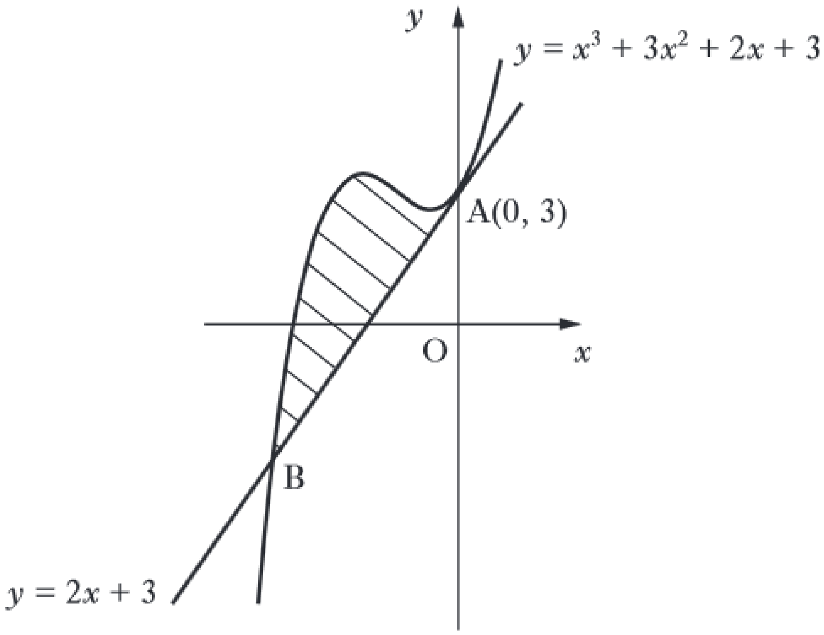
(*d*) It is also known that 55 milligrams of this serum in the body will prove fatal, and the captors wish to keep Agent 004 alive. Is there any maximum length of time for which they can continue to administer this serum and still keep him alive ?

From: SQA

8. Given the sequence $K_{n+1} = aK_n + b$. The first three terms of this sequence are 6, 12 and 21 respectively.
(i) Calculate the values of a and b.
(ii) Hence find the limit of the sequence.

Outcome 1.4: Applying calculus skills to optimisation and area

1. The line with equation $y = 2x + 3$ is a tangent to the curve with equation $y = x^3 + 3x^2 + 2x + 3$ at A(0, 3), as shown in the diagram.

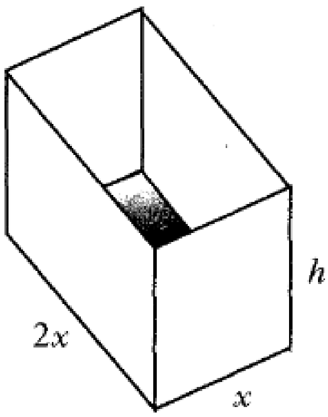


From: SQA

The line meets the curve again at B.

Show that B is the point (–3, –3) and find the area enclosed by the line and the curve.

2. An open cuboid measures internally x units by $2x$ units by h units and has an inner surface area of 12 units^2.

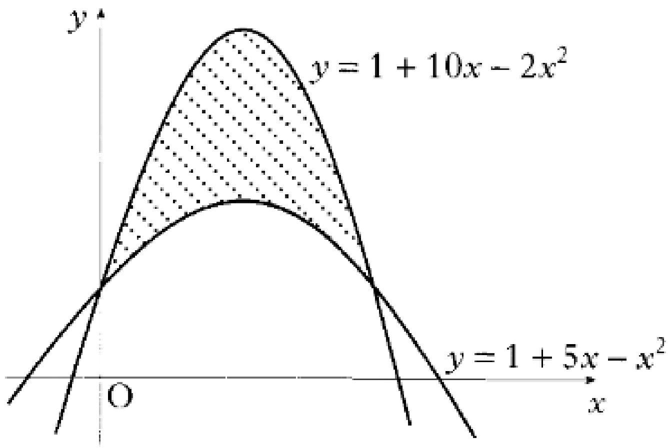


From: SQA

(a) Show that the volume, V units^3, of the cuboid is given by $V(x) = \frac{2}{3}x\left(6 - x^2\right)$.

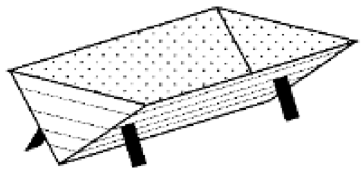
(b) Find the exact value of x for which this volume is a maximum.

3. Calculate the shaded area enclosed between the parabolas with equations $y = 1 + 10x - 2x^2$ and $y = 1 + 5x - x^2$.

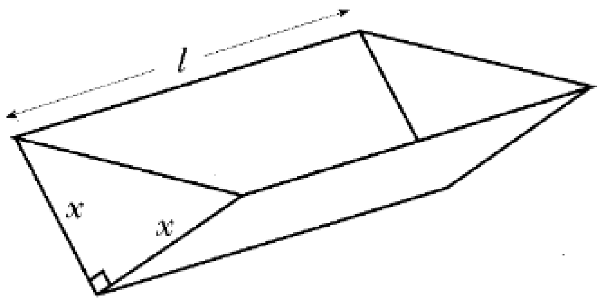


From: SQA

4. An open water tank, in the shape of a triangular prism, has a capacity of 108 litres. The tank is to be lined on the inside in order to make it watertight.



The triangular cross-section of the tank is right-angled and isosceles, with equal sides of length x cm. The tank has a length of l cm.



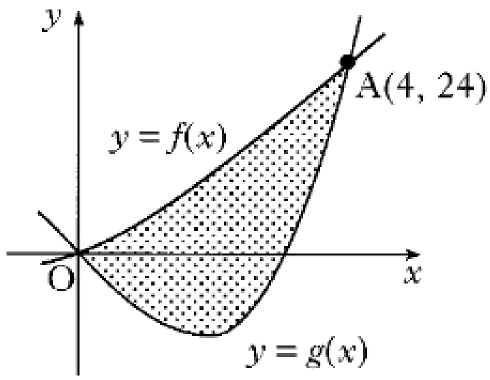
(*a*) Show that the surface area to be lined, A cm^2, is given by $A(x) = x^2 + \dfrac{432000}{x}$.

(*b*) Find the value of x which minimises this surface area.

From: SQA

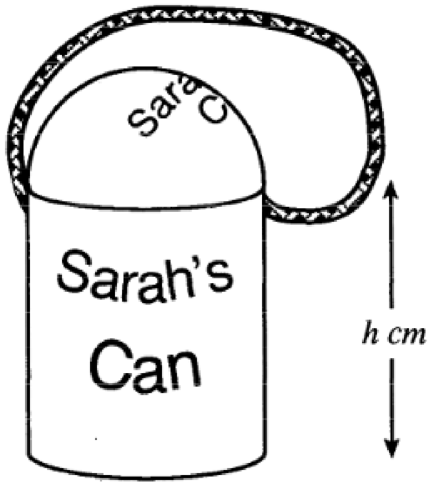
5. The incomplete graphs of $f(x) = x^2 + 2x$ and $g(x) = x^3 - x^2 - 6x$ are shown in the diagram. The graphs intersect at A(4, 24) and the origin.

Find the shaded area enclosed between the curves.



From: SQA

6. A child's drinking beaker is in the shape of a cylinder with a hemispherical lid and a circular flat base. The radius of the cylinder is r cm and the height is h cm. The volume of the cylinder is 400 cm^3.



(*a*) Show that the surface area of plastic, $A(r)$, needed to make the beaker is given by $A(r) = 3\pi r^2 + \frac{800}{r}$.

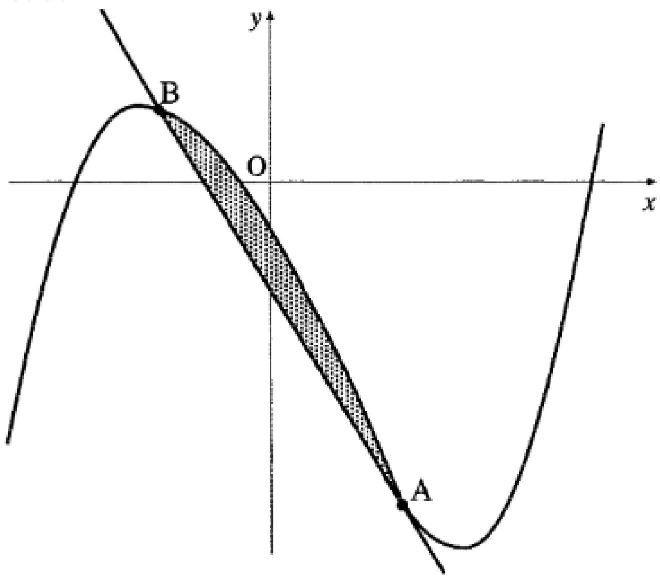
Note: The curved surface area of a hemisphere of radius r is $2\pi r^2$.

(*b*) Find the value of r which ensures that the surface area of plastic is minimised.

From: SQA Additional questions (Education Scotland)

7. In the diagram below a winding river has been modelled by the curve $y = x^3 - x^2 - 6x - 2$ and a road has been modelled by the straight line AB. The road is a tangent to the river at the point A(1, – 8).

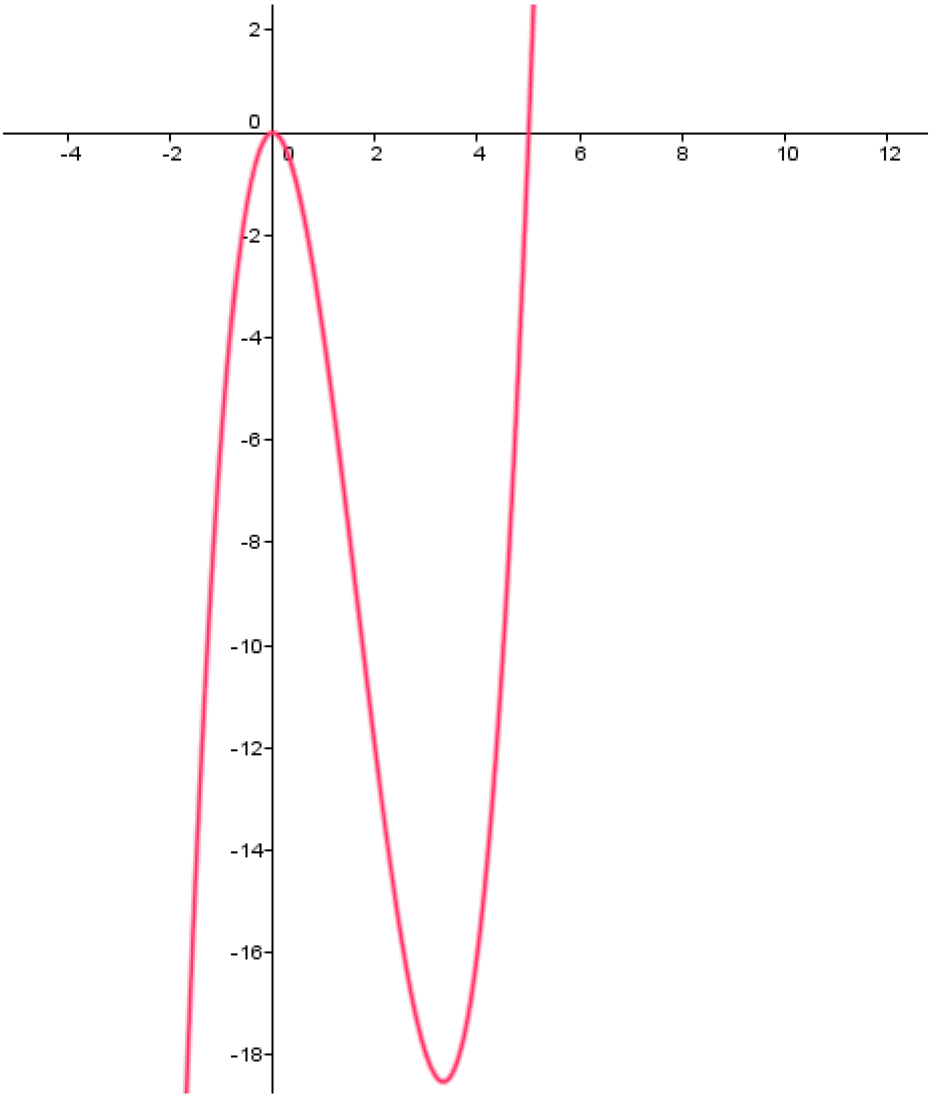
(*a*) Find the equation of the tangent at A and hence find the coordinates of B.
(*b*) Find the area of the shaded part which represents the land bounded by the river and the road.



N.B. Only complete this question if you have done Polynomials topic.

From: SQA Additional questions (Education Scotland)

8. The graph shown has the equation $y = x^2(x - 5)$. Calculate the area of the region enclosed by the curve and the x – axis.



9. The diagram shows graphs with Equations $y = -x^2 + 6$ and $y = x^2 - 2$.

(i) Write down an integral to represent the enclosed Area.

(ii) Calculate the enclosed Area.