[](http://www.google.co.uk/url?sa=i&rct=j&q=south+lanarkshire+council+logo&source=images&cd=&cad=rja&uact=8&docid=CPztyDuwoEJ_xM&tbnid=XTnaVdkSQHQfCM:&ved=0CAUQjRw&url=https://www.civica.co.uk/articles/217-South-Lanarkshire-Council-drives-efficiencies-with-case-management-upgrade-from-Civica&ei=9bngU7y9M8Ky0QWqm4HACw&bvm=bv.72197243,d.d2k&psig=AFQjCNFXmTQpeBma7hiudBmpA0AOeOIpfQ&ust=1407322984600190)



NATIONAL

QUALIFICATIONS

**MATHEMATICS**

**Advanced Higher**

**Revision Materials**

**Revision Materials**

**Applications Unit**

**Applications of Algebra and Calculus**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Standard derivatives** | |  | **Standard integrals** | |
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**Summations**

(Arithmetic series) ****

(Geometric series) ****

****

**Binomial theorem**

 where 

**Maclaurin expansion**



**De Moivre’s theorem**



**Vector product**



**Matrix Transformation**

Anti-clockwise rotation through an angle,  about the origin, ****

* 1. **Applying Algebraic skills to the binomial theorem**
* **Expanding expressions using the binomial theorem**

1. Expand the following using the Binomial Theorem:

(a)  (b)  (c)  (d)  (e) 

**1.1 Applying Algebraic skills to complex numbers**

* **Performing algebraic operations on complex numbers**

2. Express the following in the form 

(a) , ,  and , where  and 

(b) , ,  and , where  and 

(c) , ,  and , where  and 

(d) , ,  and , where ,  and 

(e) , ,  and  , where ,  and 

* 1. **Applying Algebraic skills to sequences and series**
* **Finding the general term and summing arithmetic sequences**

3. An arithmetic sequence is given by: 6, 27, 48….. Find

(a) the 40th term of the sequence (b) the sum of the first 40 terms

4. An arithmetic sequence is given by: 3, 22, 41….. Find

(a) the 30th term of the sequence (b) the sum of the first 30 terms

5. An arithmetic sequence is given by: 5, 22, 39….. Find

(a) the 50th term of the sequence (b) the sum of the first 50 terms

6. An arithmetic sequence is given by: 2, 29, 56….. Find

(a) the 40th term of the sequence (b) the sum of the first 40 terms

* 1. **Applying Algebraic skills to sequences and series**
* **Finding the general term and summing geometric sequences**

7. A geometric sequence is given by: 6, 18, 54….. Find

(a) the 15th term of the sequence (b) the sum of the first 15 terms

8. A geometric sequence is given by: 3, 6, 12….. Find

(a) the 20th term of the sequence (b) the sum of the first 20 terms

9. A geometric sequence is given by: 1, 7, 49….. Find

(a) the 11th term of the sequence (b) the sum of the first 11 terms

10. A geometric sequence is given by: 5, 20, 100….. Find

(a) the 13th term of the sequence (b) the sum of the first 13 terms

* 1. **Applying Algebraic skills to sequences and series**
* **Using the Maclaurin series expansion to find a stated number of terms of the power series for a simple function**

11. Find the first four terms of the Maclaurin series for:

(a)  (b)  (c)  (d) 

(e)  (f) 

* 1. **Applying algebraic skills to summation**
* **Know and use sums of certain series and other straightforward results**

12. Evaluate

(a)  (b)  (c)  (d) 

(e)  (f)  (g)  (h) 

* 1. **Applying summation formulae**
* **use mathematical induction to prove summation formulae**

**13.** Use proof by induction to show that, for all , ****

(a)  (b)  (c) 

(d)  (e)  (f) 

* 1. **Applying algebraic and calculus skills to properties of functions**
* **Find the asymptotes of a rational function**

**14  , **

For the graph 

1. Give the equation of the vertical asymptote.
2. Show that there is a non-vertical asymptote and state the equation.

**15  , **

For the graph 

1. Give the equation of the vertical asymptote.
2. Determine the equation of the non-vertical asymptote.

**16  , **

For the graph 

1. Write down the equation of the vertical asymptote.
2. Determine the equation of the non-vertical asymptote.

**17  , **

For the graph 

1. Write down the equation of the vertical asymptote.
2. Determine the equation of the non-vertical asymptote.

**18  , **

For the graph 

1. Write down the equation of the vertical asymptote.
2. Determine the equation of the non-vertical asymptote.

**1.4 Applying algebraic and calculus skills to properties of functions**

* **Sketch related functions**

**(Modulus or Inverse Functions, Functions Differentiated, Translations and Reflections)**

**19** Given that  , sketch the graph of  where 

**20** Given that  , sketch the graph of  where 

**21** Given that , sketch the graph of  where 

**22** Given that  , sketch the graph of  where 

**23** Let  be a function with a maximum turning point at  and a minimum turning

point at , . The graph crosses the *y*-axis at .

1. Sketch the graph  on the interval 
2. Hence sketch the graph of  on the interval 

**24** Let  be a function with a minimum turning point at  and a maximum turning

point at , . The graph crosses the *y*-axis at .

1. Sketch the graph  on the interval 
2. Hence sketch the graph of  on the interval 

**25** A particle moves in a straight line such that its displacement,  metres, from a fixed point 

on the line after  seconds, is given by

 , 

Find the velocity of the particle after 3 seconds.

**26** The equation of rectilinear motion of a particle after time,  seconds, is given by

 where  is the displacement of the particle after  seconds.

Find the velocity of this particle after 2 seconds.

**27** A particle moves a distance  metres in  seconds.

The distance travelled by the particle is given by



Find the acceleration of the particle after 4 seconds.

**28** A particle moves along a straight line such that its displacement,  metres,

from a fixed point  on the line after  seconds is given by

Find the initial acceleration of the particle.

**29** The turning effect, , of a power boat, is given by the formula

, 

where  is the angle (in radians) between the rubber and the central line of the boat.

Find the size of  which maximizes the turning effect.

**30** The net of a closed box is to be cut from a square piece of card of length 30 units.

The volume of the box formed, is given by

 where  is the length of the side of the squares to be cut from each corner.

Determine the size of the square of side  to be cut out to ensure maximum volume.

**Answers:**

**1 (a)** 

**(b)** 

**(c)** 

**(d)** 

**(e)** 

**2 (a)  , , , **

**(b)  , , , **

**(c)  , , , **

**(d)  , , , **

**(e)  ,, , **

**3 (a)  (b)  4 (a)  (b) **

**5 (a)  (b)  6 (a)  (b) **

**7 (a)  (b)  8 (a)  (b) **

**9 (a)  (b)  10 (a)  (b) **

**11 (a)  (b) **

**(c)  (d) **

**(e)  (f) **

**12 (a) 4100 (b) 7320 (c) 1110 (d) 323 (e) 171 700**

**(f) 44 280 (g) 10 497 600 (h) 6 502 500**

**13 (a)** Assume true for ** and** Consider ****

**LHS = **

**RHS = **

**LHS = RHS,** so true for ****

Using **,** Check for **, LHS =  , RHS = **

**LHS = RHS,** so true statement for **.** Hence, if true for **** implies true for

, and since true for **** then by induction, statement is true for all  .

**Answers:**

**13 (b)** Assume true for ** and** Consider ****

**LHS = **

**RHS = , LHS = RHS,** so true for ****

Using **,** Check for **, LHS =  , RHS = **

**LHS = RHS,** so statement true for **.** Hence, if true for **** implies true for

, and since true for **** then by induction, statement true for all  .

**13 (c)** Assume true for ** and** Consider ****

**LHS = **

**RHS = , LHS = RHS,** so true for ****

Using **,** Check for **, LHS =  , RHS = **

**LHS = RHS,** so statement true for **.** Hence, if true for **** implies true for

, and since true for **** then by induction, statement true for all  .

**13 (d)** Assume true for ** and** Consider ****

**LHS = **

**RHS =  LHS = RHS,** so true for ****

Using **,** Check for **, LHS =  , RHS = **

**LHS = RHS,** so statement true for **.** Hence, if true for **** implies true for

, and since true for **** then by induction, statement true for all  .

**13 (e)** Assume true for ** and** Consider ****

**LHS = **

**RHS = , LHS = RHS,** so true for ****

Using **,** Check for **, LHS =  , RHS = **

**LHS = RHS,** so statement true for **.** Hence, if true for **** implies true for

, and since true for **** then by induction, statement true for all  .

**13 (f)** Assume true for ** and** Consider ****

**LHS = **

**RHS = , LHS = RHS,** so true for ****

Using **,** Check for **, LHS =  , RHS = **

**LHS = RHS,** so statement true for **.** Hence, if true for **** implies true for

, and since true for **** then by induction, statement true for all  .

**14 (a)** vertical asymptote,  **(b)** non-vertical asymptote, 

**15 (a)** vertical asymptote,  **(b)** non-vertical asymptote, 

**16 (a)** vertical asymptote,  **(b)** non-vertical asymptote, 

**17 (a)** vertical asymptote,  **(b)** non-vertical asymptote, 

**18 (a)** vertical asymptote,  **(b)** non-vertical asymptote, 





***x***

***y***

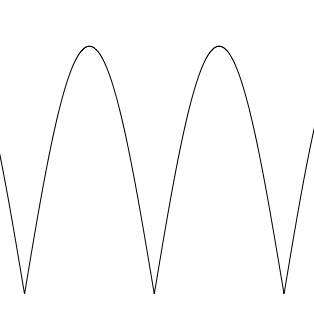
**5**

**22**

**21**

**19**

**20**



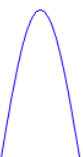
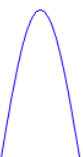
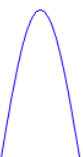
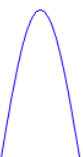




***x***

***y***

**3**

****





***x***

***y***

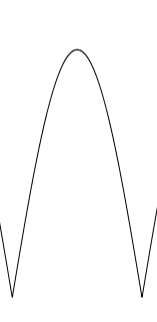




***x***

***y***

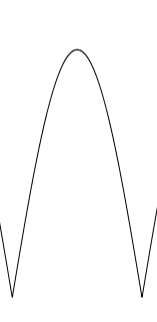
**3**

****

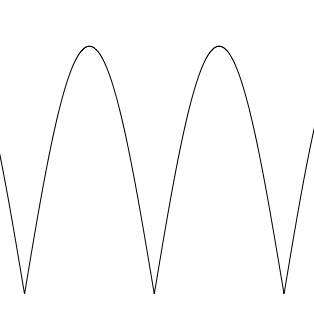
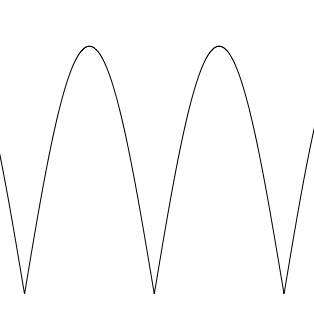
**5**

****

**1**

****

**1**



(a, 14)

***x***

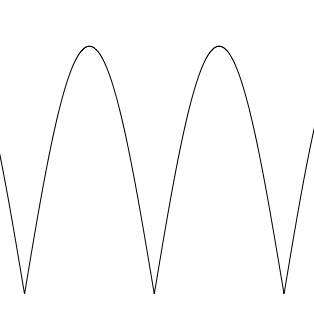
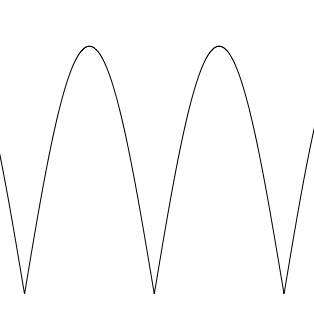
***y***

-1

(a, 11)

***x***

***y***

**23 (a) 23 (b)**

**11**

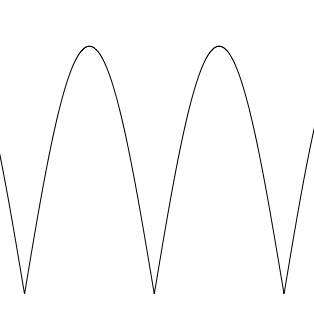
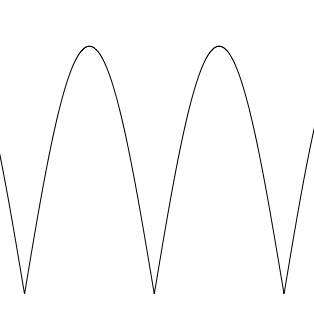
(b, 6)

**6**

**4**

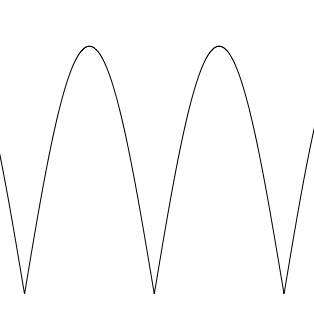
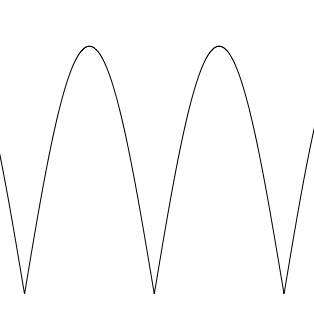
(b, -3)



***x***

***y***



***x***

***y***

**24 (a) 24 (b)**

(p, 6)

(q, 3)

(q, 5)

2

(p, -8)





**25 ,  26 , **

**27 ,  28 , **

**29 ,  30 ,** Max when ****