

X100/701

NATIONAL
QUALIFICATIONS
2009

THURSDAY, 21 MAY
1.00 PM – 4.00 PM

MATHEMATICS
ADVANCED HIGHER

Read carefully

1. Calculators may be used in this paper.
2. Candidates should answer **all** questions.
3. **Full credit will be given only where the solution contains appropriate working.**



Answer all the questions.

1. (a) Given $f(x) = (x + 1)(x - 2)^3$, obtain the values of x for which $f'(x) = 0$. 3
- (b) Calculate the gradient of the curve defined by $\frac{x^2}{y} + x = y - 5$ at the point $(3, -1)$. 4

2. Given the matrix $A = \begin{pmatrix} t + 4 & 3t \\ 3 & 5 \end{pmatrix}$.
- (a) Find A^{-1} in terms of t when A is non-singular. 3
- (b) Write down the value of t such that A is singular. 1
- (c) Given that the transpose of A is $\begin{pmatrix} 6 & 3 \\ 6 & 5 \end{pmatrix}$, find t . 1

3. Given that

$$x^2 e^y \frac{dy}{dx} = 1$$

and $y = 0$ when $x = 1$, find y in terms of x . 4

4. Prove by induction that, for all positive integers n ,

$$\sum_{r=1}^n \frac{1}{r(r+1)} = 1 - \frac{1}{n+1}. \quad 5$$

5. Show that

$$\int_{\ln \frac{3}{2}}^{\ln 2} \frac{e^x + e^{-x}}{e^x - e^{-x}} dx = \ln \frac{9}{5}. \quad 4$$

6. Express $z = \frac{(1+2i)^2}{7-i}$ in the form $a + ib$ where a and b are real numbers.

Show z on an Argand diagram and evaluate $|z|$ and $\arg(z)$. 6

7. Use the substitution $x = 2 \sin \theta$ to obtain the exact value of $\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}} dx$. 6
(Note that $\cos 2A = 1 - 2 \sin^2 A$.)
8. (a) Write down the binomial expansion of $(1+x)^5$. 1
(b) Hence show that 0.9^5 is 0.59049 . 2
9. Use integration by parts to obtain the exact value of $\int_0^1 x \tan^{-1} x^2 dx$. 5
10. Use the Euclidean algorithm to obtain the greatest common divisor of 1326 and 14654, expressing it in the form $1326a + 14654b$, where a and b are integers. 4
11. The curve $y = x^{2x^2+1}$ is defined for $x > 0$. Obtain the values of y and $\frac{dy}{dx}$ at the point where $x = 1$. 5
12. The first two terms of a geometric sequence are $a_1 = p$ and $a_2 = p^2$. Obtain expressions for S_n and S_{2n} in terms of p , where $S_k = \sum_{j=1}^k a_j$. 1,1
Given that $S_{2n} = 65S_n$ show that $p^n = 64$. 2
Given also that $a_3 = 2p$ and that $p > 0$, obtain the exact value of p and hence the value of n . 1,1
13. The function $f(x)$ is defined by
- $$f(x) = \frac{x^2 + 2x}{x^2 - 1} \quad (x \neq \pm 1).$$
- Obtain equations for the asymptotes of the graph of $f(x)$. 3
Show that $f(x)$ is a strictly decreasing function. 3
Find the coordinates of the points where the graph of $f(x)$ crosses
(i) the x -axis and
(ii) the horizontal asymptote. 2
Sketch the graph of $f(x)$, showing clearly all relevant features. 2

[Turn over for Questions 14 to 16 on Page four

14. Express $\frac{x^2 + 6x - 4}{(x + 2)^2(x - 4)}$ in partial fractions. 4

Hence, or otherwise, obtain the first three non-zero terms in the Maclaurin expansion of $\frac{x^2 + 6x - 4}{(x + 2)^2(x - 4)}$. 5

15. (a) Solve the differential equation

$$(x + 1) \frac{dy}{dx} - 3y = (x + 1)^4$$

given that $y = 16$ when $x = 1$, expressing the answer in the form $y = f(x)$. 6

- (b) Hence find the area enclosed by the graphs of $y = f(x)$, $y = (1 - x)^4$ and the x -axis. 4

16. (a) Use Gaussian elimination to solve the following system of equations

$$\begin{aligned} x + y - z &= 6 \\ 2x - 3y + 2z &= 2 \\ -5x + 2y - 4z &= 1. \end{aligned} \quad \text{5}$$

- (b) Show that the line of intersection, L , of the planes $x + y - z = 6$ and $2x - 3y + 2z = 2$ has parametric equations

$$\begin{aligned} x &= \lambda \\ y &= 4\lambda - 14 \\ z &= 5\lambda - 20. \end{aligned} \quad \text{2}$$

- (c) Find the acute angle between line L and the plane $-5x + 2y - 4z = 1$. 4

[END OF QUESTION PAPER]